Problems

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Definition 0.1. *For any vector space V, and W we get a map $V^\vee \otimes W \to \mathrm{Hom}(V, W)$. We say that $\mathcal{L} \in \mathrm{Hom}(V, W)$ is nuclear if it is in the image of this map.*

1. Show that $\mathcal{L} : V \to W$ is thick if and only if it is nuclear.
2. Show that $\mathrm{Conf}^{fin} \subset \mathrm{Conf}$ is a homotopy equivalence.
3. Show that composition is well defined in the category Q.

Definition 0.2. *If $(\mathcal{C}, \oplus)$ is a symmetric monoidal category, then define $\mathcal{C}^{-1}\mathcal{C}$ by having objects pairs $(C_+, C_-) \in \mathrm{ob}\mathcal{C}^2$ and morphisms (A, a_+, a_-) with $A \in \mathrm{ob}\mathcal{C}$, and $a_\pm : C_\pm \oplus A \to D_\pm \oplus A$.*

Definition 0.3. *Define the category Q by having objects super vector subspaces of some fixed infinitely dimensional super inner product space H. And a morphism from $V \to W$ exists only if $V \subset W$, and consists of the data of $\beta : V^\perp \to V^\perp$ an odd involution for $V^\perp \subset W$.*

d. Show that if $\mathcal{C}$ is the groupoid of finite dimensional vector spaces, then $\mathcal{C}^{-1}\mathcal{C}$ is equivalent to Q.

e. Construct a homeomorphism $BQ \to \mathrm{Conf}^{fin}$.

Theorem 0.4. *If $\mathcal{C}$ is a symmetric monoidal groupoid then $B\mathcal{C}$ is a E_∞-space. Further $B(\mathcal{C}^{-1}\mathcal{C})$ is the group completion of $B\mathcal{C}$*

vi. Argue that $BQ \simeq \mathbb{Z} \times BO$.

Riemannian bordisms need to satisfy the following condition: i_j^+ are isometric embeddings into $\Sigma - i_1(W_1^- \cup W_1^c)$.

Consider the following two 1- Riemannian manifolds, L_t and R_t below, of Length t

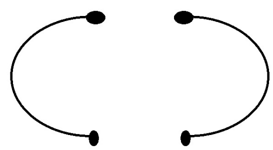

Figure 1: L_t and R_t

7. Show that R_t is a bordism for $t > 0$, and L_t is a bordism for $t \geq 0$

7. Show that as a symmetric monoidal category 1-RBord is generated by the point, R_t for $t > 0$ and L_0, subject to they are both symmetric, and $R_{t_1+t_2} = R_{t_1} \circ_{L_0} R_{t_2}$.