Problems

Jens Jakob Kjaer

January 23, 2019

1. Show that when F is $E(c, \kappa)$ then $T_1(F)$ is $E(c-1, \kappa-1)$.
2. Explain to Jens what $E(c, \kappa)$ is an analogy for in ordinary calculus.
3. When $F(X) = X^{\wedge 2}$ then $P_1(F)(X) \simeq *$.

Definition 0.1. *Let $f : F \to G$ be a natural transformation. We say that f satisfy $\mathcal{O}(c, \kappa)$ if for all X which is $k \geq \kappa$ connected, then $f_X : F(X) \to G(X)$ is $(2k - c)$ connected.*

We say that F and G agrees up to first order if there is $f : F \to G$ satisfying $\mathcal{O}(c, \kappa)$ for some c and κ.

d. Show that if $f : F \to G$ satisfy $\mathcal{O}(c, \kappa)$, then $T_1(f)$ satisfy $\mathcal{O}(c-1, \kappa-1)$

e. Show that if F and G agrees up to first order, then $P_1(F) \simeq P_1(G)$.

f. Conclude that if F and G agrees up to first order, and G is linear then $P_1(F) \simeq G$.

g. Explain to Jens what $\mathcal{O}(c, \kappa)$ is an analogy for in ordinary calculus.

Let us no longer assume that $F(*) \simeq *$. But let us take $F(*) \simeq Y$ for some space Y.

Definition 0.2. *A functor is excisive if it takes homotopy pushouts to homotopy pullbacks*

VIII. Show that if F is excisive then $\overline{F}(\ ) := \mathrm{hofib}(F(\ ) \to Y)$ is linear. Also prove the converse.

IX. Define T_1, and $E(c, \kappa)$ as before. Show that if F is stably excisive (satisfy $E(c, \kappa)$ for some c and κ) then $P_1(F) := \mathrm{colim}_i(T_1^i(F))$ is excisive.