Problems

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1. Compute ∂_3 from the definition.
2. Show that P_n commutes with finite limits, and filtered colimits
3. Show that $P_n(F \circ \Sigma) \simeq P_n(F) \circ \Sigma$ and $P_n(\Omega \circ F) \simeq \Omega P_n(F)$. Find $\partial_*(\Omega)$, and $\partial_*(\Sigma)$.
4. Show that $P_n(F \circ \mathrm{Sq}) = P_{\lfloor \frac{n}{2} \rfloor}(F) \circ \mathrm{Sq}$, where $\mathrm{Sq}(X) = X \wedge X$. Find $\partial_*(\mathrm{Sq})$.

In the following functors are between spaces and reduced. The following are some of the key observations in Sarah Yeakel's proof of the chain rule.

e. Let $\bigvee_n : \mathcal{C}^n \to \mathcal{C}$ be $(X_1, \ldots, X_n) \mapsto \bigvee_i^n X_i$. Show that

$$\mathrm{cr}_2(F)(X_1, X_2) = \mathrm{cr}_1^{(2)}\big(\mathrm{cr}_1^{(1)}\big(F \circ \bigvee_2\big)\big)(X_1, X_2),$$

where $\mathrm{cr}_1^{(i)}$ is apply the first cross effect to the i'th coordinate.

f. Show that there is a map $F \circ \mathrm{cr}_1(G) \to \mathrm{cr}_1(F \circ G)$

From here one construct maps

$$\mathrm{cr}_\mathrm{k}(F)(\mathrm{cr}_{n_1}(G), \ldots, \mathrm{cr}_{n_k}(G)) \to \mathrm{cr}_{n_1 + \ldots + n_k}(F \circ G)$$

These induces maps $\partial_k(F) \wedge \partial_{n_1}(G) \wedge \ldots \wedge \partial_{n_k}(G) \to \partial_{n_1 + \ldots + n_k}(F \circ G)$.

VII. Show that for $\Delta_2 : \mathcal{L}_2 \leftrightarrows \mathcal{H}_2 : \mathrm{cr}_2$ then $\mathrm{cr}_2 \circ \Delta_2 \simeq id$.