Problem Session 4
2/14/19

Recall that F is polynomial of degree $\leq k$ if for all $V \in \mathcal{O}(M)$ and nonempty, pairwise disjoint, closed $A_0, ..., A_k \subseteq V$, the $(k+1)$-cube

$$S \mapsto F(V - \cup_{i \in S} A_i)$$

is homotopy cartesian.

1. Show that the functor $U \mapsto \mathrm{Emb}(U, N)$ is not polynomial of degree ≤ 1.
2. Show that the functor $U \mapsto \mathrm{Map}(U^k, N)$ is polynomial of degree $\leq k$.
3. Show that if F is polynomial of degree $\leq k$ then it is also polynomial of degree $\leq k+1$.

Recall the definition of the kth derivative of F,

$$F^{(k)}(\emptyset) = \mathrm{thofiber}\left(S \mapsto F\left(\cup_{i \notin S} B_i\right)\right)$$

where the B_i are pairwise disjoint open balls in M.

4. Let $F(-) = \mathrm{Map}(-, N)$. Compute $F^{(1)}(\emptyset)$ and $F^{(2)}(\emptyset)$ from the definition. What is $F^{(k)}(\emptyset)$ in general?
5. Show that if F is polynomial of degree $\leq k$, then it has contractible derivatives of order $\geq k+1$.