

# Problems

Wednesday, March 6, 2019 11:51 AM

1) Chain rule states that the map  $\partial_x f \circ \partial_x G \longrightarrow \partial_x (f \circ G)$

Let  $\partial_x$  land in spectra; it is lax monoidal.

fact  $\partial_n$  takes fiber sequences of functors to fiber sequences of spectra.

let  $f = \text{Hom}(X, -)$ ,  $G$  a reduced finitary analytic functor.

(a) Reduce chain rule for such  $f$  to the case of  $\text{Hom}(VS^i, -)$

(b) Reduce  $\text{Hom}(VS^{i+1}, -)$  to  $\text{Hom}(VS^i, -)$

(c) Prove chain rule for  $f = \text{Hom}(VS^0, -)$   
(harder)

(d) Suppose  $f$  satisfies

$$\Gamma(x) \cong \text{colim}_{Y \in \text{Top}} F(Y) \circ \text{Hom}(x, Y) \quad (*)$$

Prove chain rule for  $f$ .

Fact We can extend the chain rule for such functions to all functions by cofibrant replacement: cofibrant functions satisfy (\*)

2) Let  $f_m(x) = \overbrace{x \circ x \circ \dots \circ x}^{m \text{ times}}$

(a) Compute  $\partial_x f_m$

(b) Compute  $(\partial_x f_m) \circ (\partial_x f_n)$

(c) Show the chain rule holds for  $f = f_m$ ,  $g = f_n$ ,  $f \circ g = f_{mn}$