Schedule for Part 2 of the Undergraduate Summer Workshop
Talks are in Hayes-Healy 127 (NOTE ROOM CHANGE)
Coffee breaks in the Math Lounge

Wednesday 7/31

1:30 - **Gabor Szekelyhidi -** Closed geodesics

2:30 - Coffee break

3:00 - **Daniel Studenmund -** Group structure from coarse geometry

4:00 - Coffee break

4:30 - **Student presentations**

6:00 - Cookout

Thursday 8/1

8:45 - 9:30 - Breakfast in Math lounge

9:30 - **Mark Behrens -** Current themes in the study of the homotopy groups of spheres

10:30 - Coffee break

11:00 - **Student Presentations**

12:00 - Lunch

1:30 - **Harrison Pugh -** Smoothing currents and de Rham's Theorem

2:30 - Coffee break

3:00 - **Eric Riedl -** Hyperbolicity and the Kobayashi Conjecture

4:00 - Coffee break

4:30 - **Student presentations**

Friday 8/2

8:45 - 9:30 - Breakfast in Math lounge

9:30 - **Stephan Stolz -** Relating topology and geometry of manifolds

10:30 - Coffee break

11:00 - **Katrina Barron -** Algebraic structures ``governed" by geometric surfaces and applications to string theory

Student Presentations

Wednesday 7/31

4:30 **Sanath Devalapurkar** - Roots of Unity in Chromatic Homotopy Theory
Ying Wang - The Inverse Problem in Equivariant (Co)homology
Robert Argus and Sabrina Traver - The Topological Structure of a Family of Excellent Morse Functions
Alvaro Pintado - Directed Homology: Theory and Computation

Thursday 8/1

11:00 **Ajmain Yamin** - Smocked Metric Spaces and their Tangent Cones
Xiayimei Han - Hermitian Symmetric Subdomains Satisfying the Griffiths Horizontality Condition
Kelley Yang - Analysis in Almost Abelian Homogeneous Spaces: Geometrical Constructions

4:30 **Sidhanth Raman** - Bounds on Entropy in Subgroups of the Braid Group
Isaac Martin - Almost Abelian Groups, their Subgroups and Automorphisms
Katie Gravel and Annie Holden - On Congruence Subgroups of the Braid Group
Alice Chudnovsky - Classifying Maps from the Braid and Symmetric Groups

Abstracts

Sanath Devalapurkar - Roots of Unity in Chromatic Homotopy Theory

In this talk, we will describe how homotopy theory places just enough structure on "homotopy rings" (E_∞-ring spectra) to prevent any analog of ramification from existing. This is via the theory of power operations. Time permitting, we will describe how equivariant homotopy theory provides a fix.

Ying Wang - The Inverse Problem in Equivariant (Co)homology

It is a natural question to ask if we can recover a space from a given cohomology. The non-equivariant case was solved by Sullivan, using the spatial realization functor. In our research, we solve the equivariant case with principal bundles and model categories.

Robert Argus and Sabrina Traver - The Topological Structure of a Family of Excellent Morse Functions

A smooth (infinitely differentiable) function f of n variables is called an excellent Morse function, if every critical point is distinct and non-degenerate (the Hessian is different from zero at this point). For a Morse function of two variables, all critical points are local maxima, local minima, or saddle points. It is not difficult to show that a Morse function on a compact domain has finitely many critical points. The topological structure of a two-variable Morse function on a compact domain can be associated with a tree, called an A-tree, whose vertices are the connected components of the level surfaces of the function. Each vertex is of degree 1 (local maximum or minimum) or 3 (saddle point). Counting the number of excellent Morse functions associated with each tree is equivalent to playing a game of "plates and olives," introduced by Liviu I. Nicolaescu, in which a plate or olive is added, combined, or removed depending on whether the associated vertex is a local maximum, local minimum, or saddle point. In this talk we discuss our research conducted this summer which involves encoding the topological structure of excellent Morse functions on the 2-sphere in tree and in an attempt to enumerate the number of successful games. That is, we consider the minimum number of ways in which a game of plates and olives can be resolved to an empty plate for a specific family of A-trees called AIN-trees.

Alvaro Pintado - Directed Homology: Theory and Computation

A directed space vaguely refers to some space with some notion of time flowing on it. Some concrete instances of directed spaces are spacetimes, directed graphs, oriented simplicial complexes, or oriented cube complexes. This presentation will motivate and introduce a theory of directed homology, suggested by existing work in homological algebra for monoids [Connes, Ionescu, Patchkoria, Street].

Ajmain Yamin - Smocked Metric Spaces and their Tangent Cones

We introduce the notion of a smocked metric spaces, explore the balls and geodesics in a collection of different smocked spaces, find their rescaled Gromov-Hausdorff limits and prove these tangent cones at infinity exist and are unique and are normed spaces. This is joint work with Prof Sormani, Dr Kazaras and a team of fellow students.

Xiayimei Han - Hermitian Symmetric Subdomains Satisfying the Griffiths Horizontality Condition

Certain homogeneous spaces known as "period domains" arise as the classifying spaces for (pure) polarized Hodge structures (PHS). The goal of the project is to identify Hermitian symmetric subdomains that satisfy a differential constraint arising in Hodge theory, namely the Griffiths horizontality condition. One should think of the PHS as an invariant associated with a smooth projective manifold, and the subdomains as parameterizing those invariants with special properties. Such subdomains are classified by "Hodge representations", so the project under considerations is essentially a question in representation theory.

Sidhanth Raman - Bounds on Entropy in Subgroups of the Braid Group

The braid group is an algebraic structure that tracks the motion of “particles” with respect to time, as they moved around in the plane, not intersecting paths. Associated to each braid is a real number called its entropy, which encodes how complicated the braid is. In this work we characterize the minimal entropy of braids in certain subgroups of the braid group. Using constructions developed by Thurston and Penner, paired with results from mapping class group theory, we proved that the minimal entropy in the Braid Torelli group and all level m congruence subgroups of the braid group have constant upper and lower bounds, independent of the number of strands. We also showed this pattern of constant upper and lower bounds doesn’t hold for all proper normal subgroups of the braid group, by constructing counterexamples from the commutator subgroup of the braid group. This is a joint work with Nia Walton, supervised by Dan Margalit and Hyunshik Shin, and supported by the NSF through the Georgia Tech Mathematics REU.

Isaac Martin - Almost Abelian Groups, their Subgroups and Automorphisms

Consider a homogeneous anisotropic model of the early universe. Such a system possesses a symmetry group that, for any fixed time, displays the full translational symmetry of Euclidean space, but changes smoothly as t is allowed to vary.

Mathematically, this system is modeled by taking a fixed homogeneous space of the given Lie group and letting the group action evolve in time following a path in the automorphism group. The automorphism group thus encodes the variety of possible dynamics. We limit our study to the class of real almost Abelian Lie groups, i.e., those real non-Abelian Lie groups which contain a codimension one Abelian subgroup. All real solvable 3-dimensional Lie groups are almost Abelian, and they appear in homogeneous cosmological models.

In this project, we first derive faithful matrix representations for all real connected almost Abelian groups and find their subgroups and homogeneous spaces. Using this representation, we then prove that the almost Abelian Lie algebra automorphisms which lift to Lie group automorphisms are exactly those acting trivially on the kernel of the exponential map, and then provide an explicit description of the automorphism groups of real connected almost Abelian Lie groups.

Kelley Yang - Analysis in Almost Abelian Homogeneous Spaces: Geometrical Constructions

In mathematical models used in sciences, a homogeneous space of a Lie group typically describes a certain system with a group of transformations. Natural structures in such a system are invariant under these transformations; therefore, the study of invariant geometrical objects on Lie groups and homogeneous spaces is indispensable in all sciences.

In this project we work with the case of almost Abelian groups. We first employ general Lie theory methods to explicitly find the left and right Haar measures, as well as the modular function, of a given such group. We proceed by studying an almost Abelian Lie group G as a manifold with a left and right G-action in order to explicitly describe left and right invariant vector and tensor fields, including Riemannian metrics and symplectic forms.

Since every almost Abelian group with an invariant Riemannian metric gives rise to a natural completely integrable Hamiltonian system, an explicit description of such invariant structures as what we have developed will lead to closed form solutions for this new class of non-compact integrable systems. It also opens doors for the study of invariant partial differential equations on these groups.

Katie Gravel and Annie Holden - On Congruence Subgroups of the Braid Group

The braid group is a mathematical object which is invaluable to math and science since it records the movement of particles through time. We study one representation of the braid group, called the symplectic representation. The kernel of the mod N reduction of the symplectic representation is known as the level N congruence subgroup, denoted $B_n[N]$. Kordek-Margalit showed a nonconstructive quartic lower bound of the size of a generating set for the level four congruence subgroup. We construct a quintic generating set for the level four congruence subgroup. Stylianakis showed that for prime ℓ, $B_n[\ell]/B_n[2\ell] \cong S_n$. We show this result can be generalized in different ways for odd and even ℓ.

Alice Chudnovsky - Classifying Maps from the Braid and Symmetric Groups

One goal of geometric group theory is to understand homomorphisms between groups. We characterized maps from the symmetric group and braid group to solvable groups, abelian groups, dihedral groups, free groups, and other groups. This process was facilitated by the concept of Totally Symmetric Sets (TSS), a “basis” of sorts for a group, developed by Margalit, Kordek and Chen. TSS induces a correspondence: given a group homomorphism $\varphi : G \longrightarrow H$ and a totally symmetric set S of size n in G, $\varphi(S)$ is either a totally symmetric set of size n in H or of size 1. By determining totally symmetric sets of the two groups, we were able to prove for many cases that all possible homomorphisms factor through a cyclic group. Moreover, for those groups, we drew conclusions on the structure of their totally symmetric sets, and gave upper bounds for their possible size.