Problem 18.
The advection terms of the Navier-Stokes equations can be written in several different forms. Although all are equivalent in continuum formulation, the discrete version can have different properties. Show that we can write
\[ u \cdot \nabla u = \frac{1}{2} \nabla (u \cdot u) - u \times \omega; \quad \text{where} \quad \omega = \nabla \times u \]

Problem 19.
Derive an expression for
\[ f_x g_y - f_y g_x \]
in a transformed coordinate system \((\xi, \eta)\).

Problem 20.
It is well known that solutions to the advection-diffusion equation
\[ U \frac{\partial f}{\partial x} = D \frac{\partial^2 f}{\partial x^2} \]
can exhibit boundary layer behavior. Assume that you want to solve this equation in a domain given by \(0 < x < 1\), that \(U > 0\), and that the boundary conditions are \(f(0) = 0\) and \(f(1) = 1\). The velocity \(U\) is high and the diffusion \(D\) is small so we expect a boundary layer near \(x = 1\).
(a) Sketch the solution for high \(U\) and low \(D\).
(b) Propose a mapping function that will cluster the grid points near the \(x = 1\) boundary.
(c) Write the equation in the mapped coordinates.