Problem 24.
The following equation (a nonlinear diffusion equation),
\[
\frac{\partial f}{\partial t} = D \frac{\partial}{\partial x} f \frac{\partial f}{\partial x} + q(x)
\]
is solved for time \(0 \leq t \leq T\) for the domain \(0 \leq x \leq L\), where \(T\) is the final time and \(L\) is the length of the domain. The source \(q(x)\), which is concentrated at \(L/2\) is turned on for a very short time at \(T/2\). For the most part we expect to be able to use a relatively coarse grid, except around the source when it is active. Thus, we want to use a grid that is refined in both time and space, around \(T/2\) and \(L/2\).

(a) Propose a mapping function to refine the grid in space and time around \(T/2\) and \(L/2\).
(b) Write down the partial differential equation in the new coordinates.

Problem 25
Show that the one fluid formulation contains the “usual” Navier-Stokes equations for the flow in each domain and the correct interface boundary conditions. The one fluid equation is:
\[
\rho \frac{\partial u}{\partial t} + \rho \nabla \cdot uu = -\nabla p + f + \nabla \cdot \mu \left( \nabla u + \nabla^T u \right) + \int_{F} \sigma \kappa \delta(x - x_f) \, da
\]
Substitute the following into this equation
\[
\begin{align*}
\mathbf{u} &= H_1 \mathbf{u}_1 + H_2 \mathbf{u}_2 \\
P &= H_1 p_1 + H_2 p_2 \\
\rho &= H_1 \rho_1 + H_2 \rho_2
\end{align*}
\]
and show that
\[
H_1 (\ldots) + H_2 (\ldots) + \delta(x_f)(\ldots) = 0
\]
=0 \quad =0 \quad =0

Problem 26
Propose a numerical scheme to solve for the unsteady flow over a rectangular cube in an unbounded domain. The Reynolds number is relatively low, 500-1000. Identify the key issues that must be addressed and propose a solution. Limit your discussion to one page. Do NOT write down the detailed finite difference equations, but state clearly what kind of spatial and temporal discretization you would use.