Problem 1. (50%)
Use the forward in time Euler method to integrate the following equation (which can arise in modeling of thin films)

\[ \frac{f_j^{n+1} - f_j^n}{\Delta t} = -U \frac{3f_j^n - 4f_{j-1}^n + f_{j-2}^n}{2h}, \quad U > 0 \]

(a) Write down the modified equation
(b) What equation is being approximated?
(c) Determine the accuracy of the scheme
(d) Use the von Neuman’s method to derive an equation for the stability conditions (you do not have to solve the equation)

Problem 2. (20%)
(a) Write down the characteristics for the following equation

\[ f \frac{\partial f}{\partial t} + (U + f) \frac{\partial f^2}{\partial x} = 0 \]

(b) Consider a sine wave initial condition \( f(x, t = 0) = \sin 2\pi x \). Sketch the characteristics and the solution at a later time.

(c) This equation can develop a shock. What is the speed of the shock?

Problem 3 (30%)
Explain what the main challenges are in solving the three different types of second order partial differential methods. For each equation type:
- Name one example of a physical situation where the equation arises;
- Explain the main challenges in solving the equation;
- Give one example of a numerical method suitable for solving it.
Limit your response to less than a page.