Problem 1. (40%)

Use the Leap-Frog time integration method to solve the diffusion equation.

\[ \frac{\partial f}{\partial t} = \alpha \frac{\partial^2 f}{\partial x^2} \quad \alpha > 0 \]

Discretize the right hand side using the standard second order centered approximation.

- Write down the discrete scheme
- Determine the accuracy of the scheme
- Use the von Neuman's method to derive an equation for the stability conditions (you do not have to solve the equation)

Hint: you can assume \( g^n = e^{n+1}/e^n = e^n/e^{n-1} \)

Problem 2. (30%)

We want to solve the unsteady advection diffusion equation in a square domain (10 by 10 units). Heat is generated by a moving heat source. The velocity is constant in the positive \( x \)-direction and is given. In order to accurately capture the solution with as coarse grid as possible, we are going to use a grid where the grid points are clustered around the source. For simplicity, the grid lines are assumed to be straight, but the clustering in \( x \) has to move with the source. The heat source starts at \((1,0.5)\) and moves with velocity \( U \) to \((9,0.5)\). The advection diffusion equation is:

\[ \frac{\partial f}{\partial t} + U \frac{\partial f}{\partial x} = D \left( \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \right) \]

(a) Sketch the domain and the grid
(b) Propose a mapping function that achieves this.
(c) Write the equation in the mapped coordinates.

Problem 3. (30%)

You have been asked to write a short proposal for a computational study of the spread of a poisonous gas in a small building, using Fluent. Assume for simplicity that that the building is a rectangle 5 by 10 by 3 meters, consisting of three rooms of nearly equal size, on one floor. The customer wants to know how you will do the simulation, how long it will take you, and what he or she will get out of it. Explain briefly:

(a) The major decisions that you need to make while setting up the problem, such as how fine the grid will be; will you use a turbulence model; is the flow steady; boundary conditions; solver type; etc. Do you need additional information? If so, what are those?

(b) How long will it take you to do the complete study, including gridding, running the computations, and analyzing the data. Give a rough breakdown of the hours.

(c) The customer has rather vague ideas about the utility of the study. What specific questions do you expect your study will be able to answer?

Limit the response to about a page.