Problem 10.
Show that the equation
\[ \frac{\partial f}{\partial t} = f \frac{\partial f}{\partial x} \]
in the domain \(-\infty < x < \infty\), with \( f \to 0 \) as \( x \to \pm\infty \), conserves the total amount of \( f \).
That is, show that the integral of \( f \) over the whole domain is independent of time.
The equation can be written in a conservative form as
\[ \frac{\partial f}{\partial t} = \frac{1}{2} \frac{\partial f^2}{\partial x} \]
Show that an "upwind" differencing of the advection term retains the conservation property of the differential equation if the conservative form is used, but not if the original form is used.

Problem 11.
Show that the Lax-Friederich flux can be written as:
\[
F_{j+1/2} = \frac{1}{2} (F_{j+1} + F_j) - \frac{1}{2} \frac{h}{\Delta t} (f_{j+1}^n - f_j^n)
\]

Problem 12.
Show that the van Leer limiter
\[
\Psi(r) = \frac{r + |r|}{1 + r}
\]
is symmetric. That is, it satisfies
\[ \frac{\Psi(r)}{r} = \Psi\left(\frac{1}{r}\right) \]