Time evolution of a one-dimensional equation

Write a program to compute the unsteady behavior of the following equation

\[
\frac{\partial f}{\partial t} + f \frac{\partial f}{\partial x} = \nu \frac{\partial^2 f}{\partial x^2}
\]

in a periodic domain of length 1 with \( \nu = 0.01 \). This is usually called the nonlinear advection-diffusion equation or the Burgers equation.

Take the initial conditions to be \( f(x,t=0) = \sin(2\pi x) + 1.0 \) and approximate the equation using a forward in time approximation for the time derivative, and centered approximations for the first and second derivative. Follow the evolution up to time 1.0 using at least three different grid resolutions, the finest of which should use at least 200 grid points. You can start from the program used in class for the linear advection-diffusion equation.

I strongly recommend that you use MATLAB, although other programming languages are also allowable. You should be able to use the program shown in class as a template. (The program is attached)

You should hand in a short description of what you did and what you see (1-2 pages), a plot of the solution at 3-4 times for one resolution, a plot of the solution at time 1.0 for at least three resolutions and a plot of the integrated difference between the solutions at that time versus (1/h). You will find that the solution is only stable if \( \Delta t \) is sufficiently small. Find, by numerical experiments, the stability limit for at least two different grid resolutions.

The project should be concise but complete and presented in the format of a short scientific paper. For a format you can use the guidelines for papers for ASME Meeting, for example:

http://www.asme.org/kb/proceedings/proceedings/author-templates

or the template for AIAA technical conferences


or any other similar format. All plots should be of publication quality and embedded in the text. For the first project, half of the points will be for presentation.
The program used in class to solve the LINEAR advection-diffusion equation:

```matlab

% one-dimensional advection-diffusion by the FTCS scheme
n=21; nstep=100; length=2.0; h=length/(n-1); dt=0.05; D=0.05;
f=zeros(n,1); y=zeros(n,1); ex=zeros(n,1); time=0.0;
for i=1:n, f(i)=0.5*sin(2*pi*h*(i-1)); end; % initial conditions
for m=1:nstep, m, time
    for i=1:n, ex(i)=exp(-4*pi*pi*D*time)*...
        0.5*sin(2*pi*(h*(i-1)-time)); end; % exact solution
    hold off; plot(f,'linewidt',2);  axis([1 n-2.0, 2.0]); % plot solution
    hold on; plot(ex,'r','linewidt',2);pause; % plot exact solution
    y=f; % store the solution
    for i=2:n-1,
        f(i)=y(i)-0.5*(dt/h)*(y(i+1)-y(i-1))+...
            D*(dt/h^2)*(y(i+1)-2*y(i)+y(i-1)); % advect by centered differences
    end;
    f(n)=y(n)-0.5*(dt/h)*(y(2)-y(n-1))+...
        D*(dt/h^2)*(y(2)-2*y(n)+y(n-1)); % do endpoints for
    f(1)=f(n); % periodic boundaries
    time=time+dt;
end;
```

% MATLAB program to produce publication quality plots.
% The data is in file example.data with the first column containing time. Here we plot the
% data in the second to fourth column versus time.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Load and plot the data
data=load('example.data');
d1=plot(data(:,1),data(:,2)); set(d1,'LineWidth',2,'LineStyle', '-', 'Color', 'k')
hold on
d2=plot(data(:,1),data(:,3)); set(d2,'LineWidth',2,'LineStyle', '--', 'Color', 'k')
d3=plot(data(:,1),data(:,4)); set(d3,'LineWidth',2,'LineStyle', '-', 'Color', 'k')
% Set the identity of each line
l1=0.5;h1=0.25; c1=plot([l1 l1+0.4],[h1 h1], '-'); set(c1,'LineWidth',2,'LineStyle', '-', 'Color', 'k'); text(l1+0.5,h1,'data1','Fontsize',18)
l2=0.5;h2=0.75; c2=plot([l2 l2+0.4],[h2 h2],'--'); set(c2,'LineWidth',2,'LineStyle', '--', 'Color', 'k'); text(l2+0.5, h2, 'data2','Fontsize',18)
l3=0.5;h3=1.25; c3=plot([l3 l3+0.4],[h3 h3], '-. '); set(c3,'LineWidth',2,'LineStyle', '-. ', 'Color', 'k'); text(l3+0.5,h3,'data3','Fontsize',18)
% Set the appearance of the plot
axis([0 5 0 5])
xlabel('Time','Fontsize',18)
ylabel('Reynolds number', 'Fontsize',18)
set(gca,'Box','on'); set(gca,'Fontsize',18, 'LineWidth',2)
hold off

% to make a eps file of the figure, use: print -deps exampleplot

Sample data file:
Example.dat

0  1  4  4
1  2  3  4
2  3  2  3
3  3  1  2
4  2  1  1
5  1  2  0