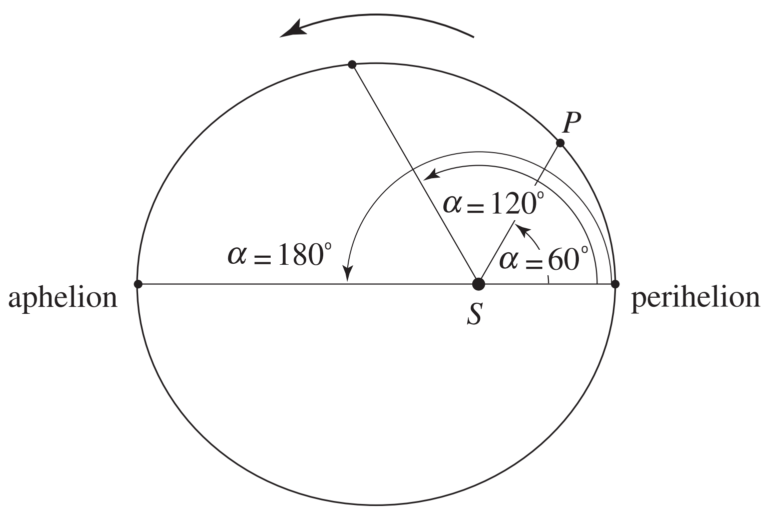
Quiz **Name**

We saw in Section 3.8 that Kepler would have been aware of the fact that the ratio $\frac{v_{\mathbf{max}}}{v_{\mathbf{min}}}$ of the maximum orbital velocity to the minimum orbital velocity is equal to $\frac{1+\varepsilon}{1-\varepsilon}$ where ε is the eccentricity of the orbit. This implies that the closer ε is to zero (the closer the orbit is to a circle) the less the variation in the velocity of the planet, and the closer ε is to one (the more elliptical the orbit is) the greater the variation in the velocity of the planet. The most circular of all the planetary orbits is that of Venus. We saw in Problem 10.31 that it traverses the three segments of its orbit from perihelion to aphelion in roughly the same time (37.03, 37.45, 37.87 days, respectively, for the first, second, and third.) See the ellipse below, but note that it is drawn more elliptically than any of the planetary orbits warrants.

Since the demotion of Pluto, the orbit of Mercury is the most elliptical of all orbits. The problem below will check the same three components for it.



1. Since one Earth year is about 365.2596 days long, it follows from the table below that the period of Mercury's orbit is about (365.2596)(0.2408) $\approx$ 87.9545 Earth days. Take α successively equal to $\frac{\pi}{3}$, $\frac{2\pi}{3}$, and π and compute the time (in Earth days) that it takes Mercury to traverse the three orbit segments depicted in the figure. Use the orbital data and formulas provided below.

Orbital Data of Planets					
Planet	semimajor axis in million km[1]	period of the orbit in years[2]	eccentricity	angle of orbital plane to Earth's	average speed in km/sec
Mercury	57.9092	0.2408	0.2056	7.00°	47.36
Venus	108.2095	0.6152	0.0068	3.39°	35.02
Earth	149.5983	1.0000	0.0167	0.00°	29.78
Mars	227.9438	1.8809	0.0934	1.85°	24.08
Jupiter	778.3408	11.8622	0.0484	1.31°	13.06
Saturn	1426.6664	29.4577	0.0557	2.49°	9.64
Uranus	2870.6582	29.4577	0.0557	2.49°	6.87
Neptune	4498.3964	29.4577	0.0557	2.49°	5.44

1) If the interest is in au, use the conversion 1 au = 149,597,892 km.
2) If the interest is in Earth days, use the conversion 1 year = 365.259636 Earth days.

Formulas:

$b = \sqrt{a^2 - c^2} \qquad \varepsilon = \frac{c}{a} \qquad \text{Area} = ab\pi \qquad \kappa = \frac{A_t}{t}$

$x = r\cos\theta,\ \ y = r\sin\theta,$

$r = a(1 - \varepsilon\cos\beta),\ \ \tan\alpha = \frac{b\sin\beta}{a(\cos\beta - \varepsilon)},\ \tan\frac{\alpha}{2} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}}\tan\frac{\beta}{2}$

$\beta - \varepsilon\sin\beta = \frac{2\pi t}{T},\ \ \beta_{i+1} = \frac{2\pi t}{T} + \varepsilon\sin(\beta_i),\ \ |\beta - \beta_i| \leq \varepsilon^i$

$v(t) = \frac{2\pi a}{T}\sqrt{\frac{2a}{r(t)} - 1}$