Quiz **Name**

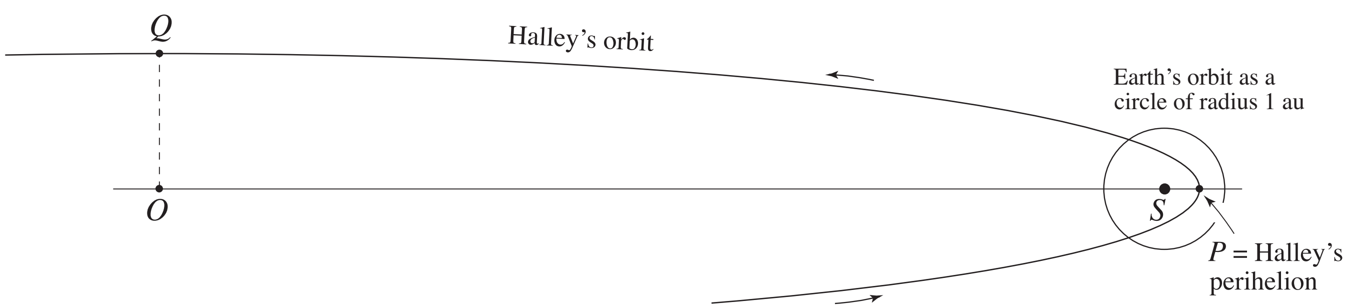
1. The comet Halley is in its orbit around the Sun. The semimajor axis of the orbit is $a = 17.83$ au, its eccentricity is $\varepsilon = 0.967$ and its period is $T = 75.32$ years. It reached its perihelion position exactly 1 year ago.

a. By successive squaring check that $\varepsilon^{32} \approx 0.3417$ and $\varepsilon^{128} \approx 0.0136$ and hence that .$\varepsilon^{160} \approx 0.0046$.

It follows that in general the expectation is that the approximation scheme $\beta_{i+1} = \frac{2\pi t}{T} + \varepsilon \sin(\beta_i)$ will require up to 160 steps to converge to $\beta(t)$ with an accuracy of two decimal places.

b. We know from Problem 10.37 of Section 10.6 that the time it takes Halley $\frac{T}{4} - \frac{T\varepsilon}{2\pi}$ to complete the first quarter of its orbit after perihelion. Check that this is equal to about 7.24 years.

c. The figure below depicts the first quarter of Halley's orbit after perihelion. The point Q marks the position of Halley at that time and S and O depict the position of the Sun and the center of the orbit. The figure includes Earth's orbit and is to scale. Use the fact that $SQ = a$ and $SO = c = \varepsilon$



together with $\cos^{-1}$ to check that $\angle QSO \approx 14.76°$ or 0.26 radians. Therefore $\angle PSQ \approx 165.24°$ or 2.88 radians.

d. Consider Halley be in the two positions H_1 and H_2 for which $\angle SPH_1 = 90°$ and $\angle SPH_2 = 120°$ and use the equations below to determine the times t_1 and t_2 (in years) it takes Halley to reach them from perihelion.

e. Compute Halley's distances from the Sun at t_1 and t_2 in au as well as its speeds in au/year. use the conversion 1 au/year $\approx$ 4.74 km/sec to express these speeds in km/sec.

Some relevant Formulas:

$$b = \sqrt{a^2 - c^2} \qquad \varepsilon = \tfrac{c}{a} \qquad \text{Area} = ab\pi \qquad \kappa = \tfrac{A_t}{t}$$

$$x = r\cos\theta,\;\; y = r\sin\theta,\;\; \tan\alpha = \tfrac{b\sin\beta}{a(\cos\beta - \varepsilon)}$$

$$r(t) = a(1 - \varepsilon\cos\beta(t)),\; \tan\tfrac{\alpha(t)}{2} = \sqrt{\tfrac{1+\varepsilon}{1-\varepsilon}}\tan\tfrac{\beta(t)}{2}$$

$$\beta(t) - \varepsilon\sin\beta(t) = \tfrac{2\pi t}{T},\;\; \beta_1 = \tfrac{2\pi t}{T},\;\; \beta_{i+1} = \tfrac{2\pi t}{T} + \varepsilon\sin(\beta_i),\;\; |\beta - \beta_i| \le \varepsilon^i$$

$$v(t) = \tfrac{2\pi a}{T}\sqrt{\tfrac{2a}{r(t)} - 1}$$