## Quiz

Name

1. Suppose that Mars in its orbit around the Sun $S$ reached its perihelion position exactly $t=200$ days ago. This assumption establishes the day as the basic unit of time. Angles are specified in radians. Use the data of the table below, work with 4 decimal accuracy, and include four decimal places in your answers.
a. Compute $\beta_{1}$.
b. Determine the angle $\beta(t)$ by finding the stable value $\beta_{i}$ that the approximation scheme $\beta_{i+1}=$ $\frac{2 \pi t}{T}+\varepsilon \sin \left(\beta_{i}\right)$ converges to.
c. Compute the corresponding angle $\alpha(t)$ and find the distance $r(t)$ in km. Locate the position of Mars on the ellipse below. (Note that Mars's orbit is more circular than depicted in the figure.)

d. What is the velocity $v(t)$ of Mars in $\mathrm{km} / \mathrm{sec}$ at that time?

| Orbital Data of Planets |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Planet | semimajor axis <br> in million $\mathrm{km}^{(1)}$ | period of the <br> orbit in years $^{(2)}$ | eccentricity | angle of orbital <br> plane to Earth's | average speed <br> in $\mathrm{km}^{\left(\mathrm{sec}^{(3)}\right.}$ |
| Mercury | 57.9092 | 0.2408 | 0.2056 | $7.00^{\circ}$ | 47.36 |
| Venus | 108.2095 | 0.6152 | 0.0068 | $3.39^{\circ}$ | 35.02 |
| Earth | 149.5983 | 1.0000 | 0.0167 | $0.00^{\circ}$ | 29.78 |
| Mars | 227.9438 | 1.8809 | 0.0934 | $1.85^{\circ}$ | 24.08 |
| Jupiter | 778.3408 | 11.8622 | 0.0484 | $1.31^{\circ}$ | 13.06 |
| Saturn | 1426.6664 | 29.4577 | 0.0557 | $2.49^{\circ}$ | 9.64 |
| Uranus | 2870.6582 | 29.4577 | 0.0557 | $2.49^{\circ}$ | 6.87 |
| Neptune | 4498.3964 | 29.4577 | 0.0557 | $2.49^{\circ}$ | 5.44 |

1) If the interest is in au, use the conversion $1 \mathrm{au}=149,597,892 \mathrm{~km}$.
2) If the interest is in Earth days, use the conversion 1 year $=365.259636$ Earth days.
3) There are $(24)(60)(60)=86,400$ seconds.

Some relevant Formulas:

$$
\begin{aligned}
& b=\sqrt{a^{2}-c^{2}} \quad \varepsilon=\frac{c}{a} \quad \text { Area }=a b \pi \quad \kappa=\frac{A_{t}}{t} \\
& x=r \cos \theta, \quad y=r \sin \theta, \quad \tan \alpha=\frac{b \sin \beta}{a(\cos \beta-\varepsilon)} \\
& r(t)=a(1-\varepsilon \cos \beta(t)), \tan \frac{\alpha(t)}{2}=\sqrt{\frac{1+\varepsilon}{1-\varepsilon}} \tan \frac{\beta(t)}{2} \\
& \beta(t)-\varepsilon \sin \beta(t)=\frac{2 \pi t}{T}, \quad \beta_{1}=\frac{2 \pi t}{T}, \quad \beta_{i+1}=\frac{2 \pi t}{T}+\varepsilon \sin \left(\beta_{i}\right), \quad\left|\beta-\beta_{i}\right| \leq \varepsilon^{i} \\
& v(t)=\frac{2 \pi a}{T} \sqrt{\frac{2 a}{r(t)}-1}
\end{aligned}
$$

