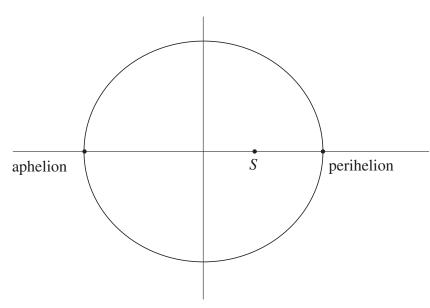
- 1. Suppose that Mars in its orbit around the Sun S reached its perihelion position exactly t=200 days ago. This assumption establishes the day as the basic unit of time. Angles are specified in radians. Use the data of the table below, work with 4 decimal accuracy, and include four decimal places in your answers.
- **a.** Compute β_1 .
- **b.** Determine the angle $\beta(t)$ by finding the stable value β_i that the approximation scheme $\beta_{i+1} = \frac{2\pi t}{T} + \varepsilon \sin(\beta_i)$ converges to.

c. Compute the corresponding angle $\alpha(t)$ and find the distance r(t) in km. Locate the position of Mars on the ellipse below. (Note that Mars's orbit is more circular than depicted in the figure.)



d. What is the velocity v(t) of Mars in km/sec at that time?

| Orbital Data of Planets | | | | | |
|-------------------------|--------------------------------|-------------------------|--------------|------------------|-------------------|
| Planet | semimajor axis | period of the | eccentricity | angle of orbital | average speed |
| | in million $\mathrm{km}^{(1)}$ | orbit in years $^{(2)}$ | | plane to Earth's | in $km/sec^{(3)}$ |
| Mercury | 57.9092 | 0.2408 | 0.2056 | 7.00° | 47.36 |
| Venus | 108.2095 | 0.6152 | 0.0068 | 3.39° | 35.02 |
| Earth | 149.5983 | 1.0000 | 0.0167 | 0.00° | 29.78 |
| Mars | 227.9438 | 1.8809 | 0.0934 | 1.85° | 24.08 |
| Jupiter | 778.3408 | 11.8622 | 0.0484 | 1.31° | 13.06 |
| Saturn | 1426.6664 | 29.4577 | 0.0557 | 2.49° | 9.64 |
| Uranus | 2870.6582 | 29.4577 | 0.0557 | 2.49° | 6.87 |
| Neptune | 4498.3964 | 29.4577 | 0.0557 | 2.49° | 5.44 |

- 1) If the interest is in au, use the conversion 1 au = 149,597,892 km.
- 2) If the interest is in Earth days, use the conversion 1 year = 365.259636 Earth days.
- 3) There are (24)(60)(60) = 86,400 seconds.

Some relevant Formulas:

$$b = \sqrt{a^2 - c^2} \qquad \varepsilon = \frac{c}{a} \qquad \text{Area} = ab\pi \qquad \kappa = \frac{A_t}{t}$$

$$x = r\cos\theta, \quad y = r\sin\theta, \quad \tan\alpha = \frac{b\sin\beta}{a(\cos\beta - \varepsilon)}$$

$$r(t) = a(1 - \varepsilon\cos\beta(t)), \quad \tan\frac{\alpha(t)}{2} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}}\tan\frac{\beta(t)}{2}$$

$$\beta(t) - \varepsilon\sin\beta(t) = \frac{2\pi t}{T}, \quad \beta_1 = \frac{2\pi t}{T}, \quad \beta_{i+1} = \frac{2\pi t}{T} + \varepsilon\sin(\beta_i), \quad |\beta - \beta_i| \le \varepsilon^i$$

$$v(t) = \frac{2\pi a}{T}\sqrt{\frac{2a}{r(t)} - 1}$$