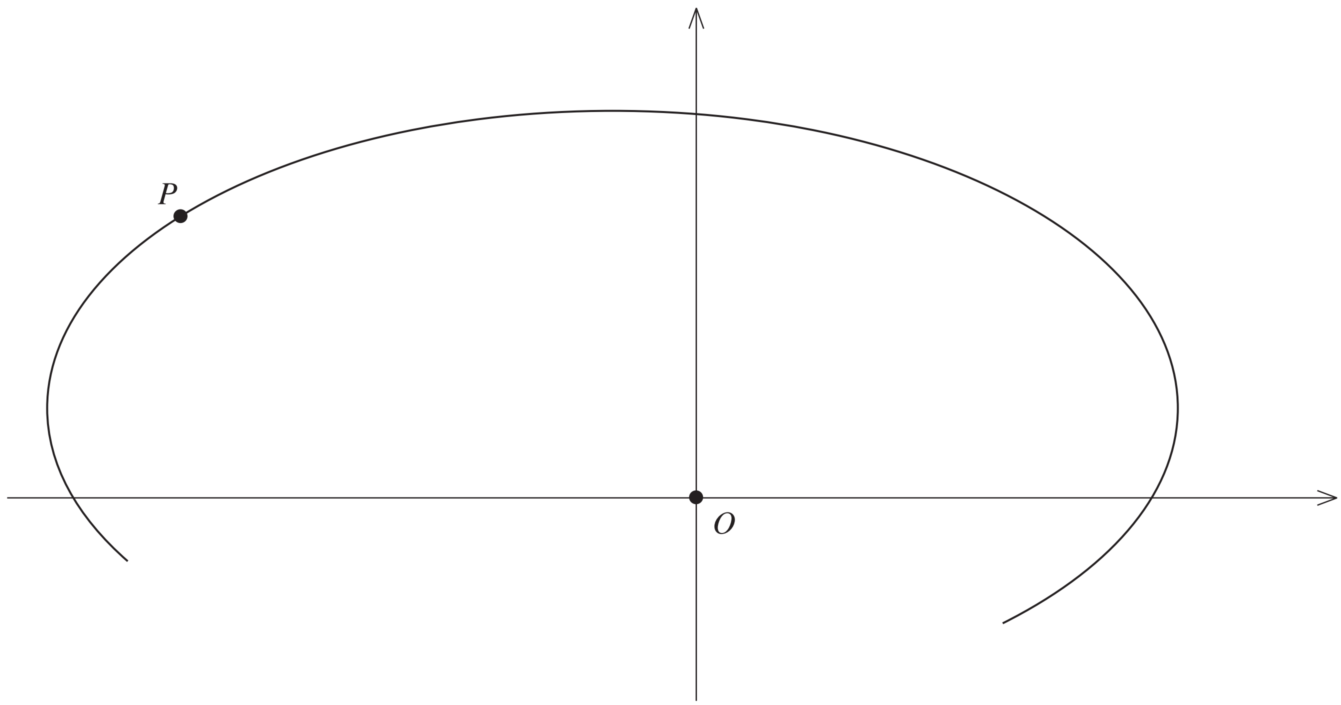
**Quiz** **Name**

The diagram below shows an $x$-$y$-coordinate system with a polar coordinate system superimposed. The trajectory of a planet $P$ is shown, and the origin $O$ is also the center of force $S$. Put $P$ (at elapsed time $t$) in typical position into the diagram. Place the corresponding $x(t), y(t), r(t), \theta(t)$ and $F(t)$ into the diagram.




You are given that $\frac{d}{dt}(r^2\frac{d\theta}{dt}) = 2r\frac{dr}{dt}\cdot\frac{d\theta}{dt} + r^2\cdot\frac{d^2\theta}{dt^2} = 0$. Starting with $\frac{dx}{dt} = \frac{dr}{dt}\cos\theta - r\sin\theta\cdot\frac{d\theta}{dt}$, verify that $\frac{d^2x}{dt^2} = \cos\theta\big[\frac{d^2r}{dt^2} - r\cdot(\frac{d\theta}{dt})^2\big]$. This equation and the analogous equation $\frac{d^2y}{dt^2} = \sin\theta\big[\frac{d^2r}{dt^2} - r\cdot(\frac{d\theta}{dt})^2\big]$ are the key ingredients in the verification of the polar force equation $F(t) = m\big[\frac{4\kappa^2}{r(t)^3} - \frac{d^2r}{dt^2}\big]$.

Formulas and Facts:

Formulae: $F_P = C_P m \frac{1}{r^2}$ $\quad M = \frac{4\pi^2 a^3}{GT^2}$ $\quad \frac{d}{dx} a^x = \ln a \cdot a^x$

$\log_a x = \frac{1}{\ln a} \cdot \ln x$ $\quad A = \displaystyle\int_\alpha^\beta \frac{1}{2} f(\theta)^2 d\theta$

$r(t)^2 \cdot \theta'(t) = c$

$x \cdot \frac{dy}{dt} - y \cdot \frac{dx}{dt}$ $\quad 2\frac{dr}{dt} \cdot \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{dt^2}$

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx} \tan^{-1} x = \frac{1}{x^2+1}$

length of a graph $= \int_a^b \sqrt{1 + f'(x)^2}\, dx$