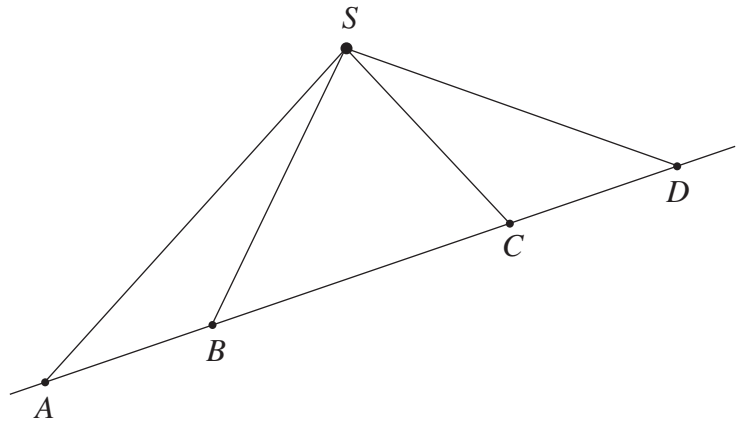


Quiz

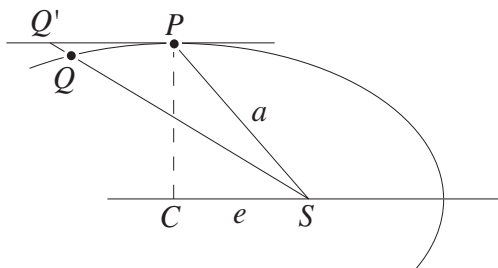
Name

1a. In the diagram below, the points  $A$ ,  $B$ ,  $C$ , and  $D$  lie on a straight line and the distance between  $A$  and  $B$  is equal to the distance between  $C$  and  $D$ . Use the diagram to explain why the areas of the triangles  $\triangle ABS$  and  $\triangle CDS$  are equal.



1b. Suppose that a point mass  $P$  of mass  $m$  moves along a straight line with constant speed. Let  $S$  be any point not on the line. Show that the segment  $PS$  sweeps out equal areas in equal times.

2. The figure below considers a planet in position  $P$  at the “top” of its elliptical orbit and again at a time  $\Delta t$  later at  $Q$ . The point  $C$  is the center of the ellipse. For a small  $\Delta t$ ,  $PQ$  is approximately equal to  $PQ'$  and the area of the elliptical sector  $SPQ$  is approximately equal to the area of the triangle  $SPQ'$ .



a) Let  $\Delta s = Q'P$ . Determine an expression involving Kepler’s constant  $\kappa$  and the semiminor axis of the ellipse that approximates the average speed  $\frac{\Delta s}{\Delta t}$  of the planet from  $P$  to  $Q$ .

b) Use the fact that  $\kappa = \frac{ab\pi}{T}$  and push  $\Delta t$  to zero to show that the speed of the planet at  $P$  is equal to  $v_P = \frac{2\pi a}{T}$ .

Formulas and Data:  $A = ab\pi$ ,  $a^2 = b^2 + c^2$ ,  $\varepsilon = \frac{c}{a}$ ,  $F = ma$ ,  $\kappa = \frac{A_t}{t}$ ,  $M = \frac{4\pi^2 a^3}{GT^2}$ ,  
 $G = 6.673 \times 10^{-11}$  in M.K.S.,  $F = G\frac{mM}{r^2}$ ,  $F = \frac{4\pi^2 a^3 m}{T^2} \frac{1}{r_P^2}$ ,  $\frac{a^3}{T^2} = \frac{GM}{4\pi^2}$ ,