Quiz **Name**

Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ with semimajor axis a, semiminor axis b and eccentricity $\varepsilon = \frac{c}{a}$ where $c = \sqrt{a^2 - b^2}$.

1. Show that the upper half of the ellipse is the graph of the function $f(x) = \frac{b}{a}(a^2 - x^2)^{\frac{1}{2}}$.

2. Show that the definite integral for the surface area S of the solid obtained by revolving of upper half of the ellipse once around the x-axis is equal to

$$S = 2\pi \int_{-a}^{a} f(x)\sqrt{1 + f'(x)^2}\, dx \;=\; \tfrac{2\pi b}{a} \int_{-a}^{a} \sqrt{a^2 - \varepsilon^2 x^2}\, dx.$$

3. Assume that $\varepsilon \neq 0$ (what happens if $\varepsilon = 0$?). Use the substitution $x = \frac{a}{\varepsilon}\sin\theta$ with $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and the formula $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$ to show that

$$\int_{-a}^{a} \sqrt{a^2 - \varepsilon^2 x^2}\, dx = \varepsilon \int_{-a}^{a} \sqrt{(\tfrac{a}{\varepsilon})^2 - x^2}\, dx \;=\; \tfrac{a^2}{\varepsilon} \int_{-\alpha}^{\alpha} \cos^2\theta\, d\theta \;=\; \tfrac{a^2}{2\varepsilon}(2\alpha + \sin 2\alpha)$$

where $\alpha = \sin^{-1}\varepsilon$. (To see that the lower limit is $-\alpha$ use the fact that $\sin^{-1}(-\varepsilon) = -\sin^{-1}\varepsilon = -\alpha$. This follows from the trig formula $\sin(-\theta) = -\sin\theta$ and the function/inverse relationship of Section 9.9.)

4. Take $a = 5$ and $b = 4$. So $c = 3$ and $\varepsilon = 0.6$. Use a calculator to show that in this case, the surface area S is approximately equal to 235.31. Assess the reasonableness of this answer by computing the area of the surface of the cylinder—without the two discs at its ends—of height $2a = 10$ and radius $b = 4$ that contains the elliptical solid.)