Section 1.1. The non-measurability proof of the hypotenuse $h$ does appear to be the same as that of the Pythagoreans. See page 91 of [1]. The extent to which the crisis brought on by the discovery of the irrationality of $\sqrt{2}$ contributed to the downfall of the Pythagoreans is not clear. See pages 47-48 of [11] for example. The parenthetical comment that the construction of the real number system "did not occur until the 16th century" oversimplifies the matter. There are two different issues to consider. One is the use of real numbers and an efficient notation for them. The other is the rigorous construction of the system of real numbers. The use of real numbers in decimal notation was advocated by Francois Vieta and Simon Stevin in the latter part of 16th century, and by John Napier in the early part of the 17th. An initial glimpse of the foundation of the real number
system can be seen in the theory of proportion of Eudoxus (roughly 408-355 B.C.). Euclid presents this theory in Book V of the *Elements*. The rigorous construction of the system of real numbers is the work of 19th century, notably that of Richard Dedekind and Georg Cantor. See [9] and [10] for details.

Section 1.3. This discussion is based on [5]. This article also provides an interesting historical perspective that draws into question the traditional interpretation of Erathosthenes’s measurement. This, see [2] pages 106-109, uses \( \varepsilon = \frac{360}{50} = 7\frac{1}{5}^\circ \) and the relationship 1 mile = \( \frac{252,000}{24,662} \approx 10.22 \) stades to conclude that the value for the radius of the Earth that Eratosthenes achieves is the equivalent of 3925 miles.

Section 1.4. Greek trigonometry has a different flavor from the modern version presented here. See [2], Chapter 17. The discussion of the limit processes \( \lim_{\theta \to 0} \sin \theta = 0 \) and \( \lim_{\phi \to \pi/2} \sin \phi = 1 \) are improved if Figure 1.21 is embedded in a circle of radius 1 and center \( C \). This way it is clear that as the right triangle shrinks, the hypothenuse is fixed and equal to 1, \( a \) shrinks to 0, and \( b \) expands to 1. See the figure above.

Section 1.5. The sides of the "cosmic" triangles of this section that have the Earth as an endpoint are all curved because the Earth’s atmosphere bends light rays. This phenomenon, called refraction, is a flaw in Aristarchus’s argument. See Exercises 9J at the end of Chapter 9 for more information about refraction.

Section 1.6. Truer to the spirit of Archimedes’s analysis is the following modification of the discussion. Take \( r_E \leq 50,000 \) miles. So \( r_S \leq 1.5 \times 10^6 \) miles. Hence \( D_S < \frac{(60)(50,000)(400)}{(66)(55,000)} < 4 \times 10^8 \) miles. The last speculation of Archimedes can now be modified to \( D_* < 3 \times 10^{12} \) miles.

It is the purpose of Section 1.6 to meet Archimedes, to give a glimpse of the grand and daring way in which he thought, and to mention his commentary about the studies of Aristarchus. Given that this is its goal, the discussion that starts on page 19 (left column) with ”Our description ... ” and concludes on page 21 (left column) with ” ... enough to fill the sphere of the universe.” is distracting in the amount of detail that it provides. Because this material adds little of mathematical relevance, it is best replaced by a concise summary to the following effect:

"By a strange combination of wild speculations (that have Aristarchus’s estimates as starting point),
a careful measurement of the angular diameter of the Sun, and lots of very delicate geometric analysis, Archimedes surmised the distance from the Earth to the sphere of fixed stars to be no larger than about $3 \times 10^{12}$ miles. By counting the number of grains of sand in spheres of larger and larger size - he starts with a sphere the size of a poppy seed, moves to one of diameter of one finger breadth, and so on - he announces a staggering $10^{63}$ grains of sand to be more than enough to fill the sphere of fixed stars.”

Section 1.7. The phenomenon of stellar parallax is also of considerable historical significance from the 15th century forward. Is the Earth the fixed point of the planetary system (and also the universe) or is the Sun the fixed point around which the planets (from the Greek ”planetes” or wanderers) including the Earth move? Would not our observed picture of the heavens change if the Earth moved around under the canopy of stars? The fact that no such change was observed and that the firmament of stars looked identical from month to month, was a credible argument against the motion of the Earth. We now know that the distances involved are so vast that the changes in the relative positions of the stars are almost imperceptible. No one was able to observe and measure the parallax of stars until the middle of the 19th century, when Bessel was first able to do so.

The discussion about stellar parallax on page 22 can be improved as follows: Denote the distance $D_A$ in AU to the star $A$ by $D_{AU}$. So

$$p_{rad} = \frac{1}{D_{AU}}.$$ 

In the current context it is customary to express angles in seconds and distances in light years. So let $p_{sec}$ be the angle of parallax in seconds and $D_{LY}$ the distance to the star in light years. A more careful computation (than the one given in the text) shows that

$$1 \text{ radian} = \frac{180}{\pi} \cdot 3600 \approx 2.06 \times 10^5 \text{ seconds}.$$ 

Also more precisely as in the text, $1 \text{ LY} = (186 \times 10^3)(31.6 \times 10^9) \approx 5.87 \times 10^{12}$ miles, so that

$$1 \text{ LY} \approx \frac{5.87 \times 10^{12}}{9.30 \times 10^9} \approx 6.31 \times 10^4 \text{ AU}.$$ 

Converting seconds back into radians and LYs back into AUs, we get

$$p_{rad} \approx p_{sec} \times \frac{1}{2.06 \times 10^5} \quad \text{and} \quad D_{AU} = 6.31 \times 10^4 D_{LY}.$$ 

So $p_{sec} \times \frac{1}{2.06 \times 10^5} \approx \frac{1}{6.31 \times 10^4D_{LY}}$, and therefore

$$p_{sec} \approx \frac{2.06 \times 10^5}{6.31 \times 10^4D_{LY}} \approx \frac{3.26}{D_{LY}}.$$ 

So $D_{LY} \approx \frac{3.26}{p_{sec}}$. 

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