About the Computations in this Text

It most situations the numerical data that describe a physical situation are only approximate. This is so because measurements of physical quantities such as length, time, area, and volume are necessarily limited in terms of their precision. For instance, a metal rod of a delicate instrument will never be precisely, say, 3 inches long. It might be 3.00000001 inches long, or 2.999999999994 inches long, but not precisely 3 inches long. Clearly, the results of computations with numbers that are only approximations will only be approximations themselves. How good are these resulting approximations? How confident can we be in them? The standard strategies with which such questions are addressed are briefly descibed below.

Significant Figures. Consider a number. If it has a decimal point, then count the leftmost non-zero digit and all the digits to the right of the leftmost non-zero digit. If it does not have a decimal point, count the leftmost non-zero digit and all the digits to its right up to and including the rightmost non-zero digit. This count is the number of *significant figures* of the number. For example, the numbers 32748, 46.290, 6.0000, 5675200, and 0.0018050 all have five significant figures.

Rounding Off. There is a *roundoff* procedure by which a number can be converted to another number of essentially the same size, but with fewer significant figures. The essence is to keep a required number of significant figures and to replace any digit beyond that number by a zero, except if it falls to the right of the decimal point, in which case it is deleted altogether. This is the essence of the matter, but there is an exception! Before changing the last required digit to zero or dropping it, have a good look at it. If it is 5 or greater, then increase the digit to its left by 1. The rounding off procedure just described is best illustrated by examples. Let's round off the five numbers above to three significant figures. Start with 32748. There are no decimal points, so the 8 is replaced by 0 and the 4 is replaced by 0. Because 4 is less than 5, the 7 remains a 7. So we get 32700. Now to 5675200. Change 2 to 0 and, before changing 5 to 0, increase 7 to 8. So the required number is 5680000. What about 46.290? Both 0 and 9 are dropped, and the 2 is increased to 3. So the number is 46.3. In the same way, 6.0000 changes to 6.00 and 0.0018050 to 0.00181. Notice that all the new numbers have three significant figures.

Computational Strategies. Consider the following example. You are informed that a hallway in a building has a width w of 9 feet and a length l of 51 feet. So the area of the hallway is given by $wl = 9 \cdot 51 = 459$ feet^2. No further information being given, we can only assume that the two dimensions are approximations and hence that the computed area of 459 feet^2 is an approximation as well. To what extent can one have confidence in this number? One way to proceed is to: count the number of significant figures of each of the numbers that is supplied; consider the smallest of these significant figures; and - after carrying out the required computation with, say, a calculator - round off the answer to that number of significant figures. In the situation above, the data are 9 and 51, so the smallest number of significant figures is *one*. Rounding off the answer of 459 feet^2 to one significant figure provides the final answer of 500 feet^2. As a consequence, the understanding is that the area is somewhere between 450 feet^2 and 550 feet^2. A different approach is this. Given the certainty that $8.5 \leq w < 9.5$ and $50.5 \leq l < 51.5$ it follows that $429.25 \text{ feet}^2 = 8.5 \cdot 50.5 \leq wl < 9.5 \cdot 51.5 = 489.25 \text{ feet}^2$ is certain as well. So the earlier strategy of rounding off to one significant figure was too conservative. Rounding 459 to 460 (two significant figures) would have given a better result. In particular, the standard roundoff strategy is not always optimal.

In this Text. Because the mathematical exposition of Part I of the text follows a historical flow, the computations will usually follow the historical spirit and not the strategies discussed above. A typical example is the discussion in Chapter 1 of Aristarchus's estimates of the sizes of the Earth, Moon, and Sun and the distances between them. In Part II there is more of an effort to be accurate, but the primary focus will be on the mathematical procedures and their applications rather than on numerical accuracy. In some situations accuracy is paid attention to. For example, in the context of the financial situations of Chapter 12, the strategy is to compute with the full accuracy that a typical hand calculator provides, and then to round off to the nearest cent. In situations where accuracy is an issue, the following notational strategy is often used. Suppose, for example, that x is a number whose correct decimal expansion up to four decimal places is, say 5.3896. Most of the time, this will expressed as $x = 5.3896$. If the value of x is not equal to 5.3896 "on the nose" and if we wish to make the point that it is only an approximation, then we will instead write $x \approx 5.3896$, where $\approx$ means approximately equal to.