Rotational Inertia, Angular Acceleration, and Frictional Torque

Introduction: Today’s apparatus consists of a string that connects a falling mass and a brass disk. The falling mass spins the disk so that the mass and disk to accelerate together. You will calculate the acceleration in three studies.

I. Measure the velocity of the falling mass using two photogates.
II. Measure the period of the rotating disk to calculate the tangential velocity/acceleration.
III. Use Force Diagrams to calculate the theoretical acceleration.

Study I: Measurements of the Falling Mass
In Study I, you will also test the principles of conservation of energy for a system with rotational and translational motion. NOTE: The disk is free to rotate, but the bearings are not frictionless.

I. Calibrate Data Studio to Measure Velocities 1 & 2
1. Referring to Figure 1 fill out the parameters listed in Table 1.
   \[ m = \text{mass of the aluminum “flag” (falling mass)} \]
   \[ L = \text{length of the “flag”} \]
   \[ y_f - y_i = \text{distance between photogates (greater than 1 m)} \]

2. Open Data Studio and double click on Digital Plug 1. Choose Photogate. Under the Measurements tab select Velocity in Ch. 1, and under the Constants tab type in the length L of your falling mass IN METERS.

3. Double click on the Plug 2 icon and repeat step 3 (re-entering values for Photogate 2).

4. Drag the Table icon to Velocity in Gate ch. 1. Do the same for Velocity in Gate ch. 2. Your Table should have 4 columns: t1, v1, t2, and v2.

Table I: Parameter Data Values (Use MKS units (meters, kg, s) for this experiment)

<table>
<thead>
<tr>
<th>( M_{\text{disk}} ) (stamped)</th>
<th>( R_{\text{disk}} ) = Radius of the Disk</th>
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<table>
<thead>
<tr>
<th>( m_{\text{flag}} ) (kg)</th>
<th>( y_f - y_i ) (m)</th>
<th>( \delta(y_f - y_i) ) (m)</th>
<th>( L ) (m)</th>
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From your parameter table, calculate the change in potential energy of the falling mass:
\[ \Delta U = mg (y_f - y_i) \text{ and } \delta(\Delta U) = mg \delta(y_f - y_i) \]
II. Collect Data
1. Wind up the cord around the brass disk – make sure to hook the string around the post on the disk. Position the small mass so that it hangs above the upper photogate.

2. Click Start, release the mass, and let it fall to the ground. As it falls, the velocities \( v_i \) and \( v_f \) of the falling mass will be recorded as it passes through each photogate. The time to fall between photogates \( t_{1to2} \) will also be measured.

3. When the weight has hit the floor, click on Stop. At this point, be sure that your measurements only include the mass falling while suspended on the string. Be sure there is no free fall or bouncing!

4. Perform the experiment 5 times. Record the initial and final velocity of each run along with the time \( t_{1to2} \) into Data Table Two (because of some quirks in measurements, you will probably have extra numbers and blanks. It should be obvious which values represent true velocities). You should have reasonably similar numbers in all runs.

<table>
<thead>
<tr>
<th>Table II: Velocity Measurements and Kinetic Energy Calculations</th>
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<tbody>
<tr>
<td>Run</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
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</table>

III. Calculate Acceleration of the Falling Mass
1. Using manual columns, enter your time and velocity data in Graphical Analysis.

2. Using calculated columns, calculate acceleration \( a = \frac{v_f^2 - v_i^2}{t_{2-1}} \) of each run.

3. Calculate the average and standard deviation using the STAT button. Calculate the error using \( \delta a = \sigma(a)/\sqrt{N} \).

\[ a = \text{____________} \pm \text{____________} \]

IV. Conservation of Energy \( \Delta E = \Delta K + \Delta U \)
You've already calculated the change potential energy is given by the vertical location of the falling mass: \( \Delta U = mg\Delta y \)

At any given time, the kinetic energy of the system is given as: \( K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \)

where \( I = 1/2MR^2 \), and \( \omega = v/R \), \( v \) is the velocity of the falling mass, \( \omega \) is the angular velocity of the disk, and \( R \) the radius of the disk. Kinetic energy can be simplified to:

\[ \Delta K = 1/2 \ast (m + M/2) \ast (v_f^2 - v_i^2) \]
Is it acceptable to neglect friction in this experiment? To test this, determine if your measurements show that mechanical energy is conserved within error. If total mechanical energy is conserved, and the system is assumed to be frictionless, then within error, a measurement of \( \Delta E = \Delta K + \Delta U \) should be consistent with zero.

1. Use GA to calculate \( \Delta K = \frac{1}{2} \left( m + \frac{M}{2} \right) \left( v_f^2 - v_i^2 \right) \) for each run.

2. Calculate the average \( \Delta K \) and standard deviation with STAT. Calculate \( \delta \Delta K \)

\[
\Delta K \quad \pm \quad \text{__________}
\]

3. Calculate \( \Delta E \) and \( \delta \Delta E = \sqrt{\left( \frac{\delta \Delta K}{\Delta K} \right)^2 + \left( \frac{\delta \Delta U}{\Delta U} \right)^2} \)

4. Are you consistent with zero? If not, go back and think about where you may have gone wrong in your measurements/assumptions.

**Study II: The Spinning Disk**

What is the tangential acceleration brass disk as the mass falls? What is the deceleration due to friction? You'll drop the mass once while a photogate measures the disk.

**I. Calibration and Measurement**

1. In DataStudio select File->New. Double-click the Digital Plug in ch. 3 and select “Photogate and Pendulum”. Check **Period (s)** and uncheck **Velocity (m/s)**.

2. Make a graph to plot Elapsed time versus **Period**. Drag the **Table** icon up to **Period**.

3. Wind up the cord of the small aluminum mass around the brass disk.

4. Hit **Start**, release the aluminum mass and let it fall to the ground. Wait until you have about 100 rows or so of data recorded. Then STOP Data Studio.

5. If you don't have two rows of data in your table (Elapsed time and Period), click on the small clock face icon. You will now have 2 numbers in each box of the table. Copy both columns.

**II. Data Analysis**

1. Return to Graphical Analysis. Under File, select New. Paste the data into the new data table. The 2 columns will be Elapsed Time in seconds and Period in seconds. Label these appropriately.
2. Make a new column to calculate the disk’s tangential velocity \( v = 2\pi R_{\text{disk}}/T \). Where T is the period of rotation.

3. Graph Velocity vs. Elapsed Time. Your graph should have two distinctly different sections. Fit each with a linear fit (adding errors). The slope of the fit is the tangential acceleration of the disk. Record each acceleration, labeling them appropriately.

4. Print a copy of the graph for each group member.

**Study III: Force Diagrams for the Mass/Disk System**

Using force diagrams, the theoretical acceleration of the system can be calculated. As the small mass falls, the following expressions are relevant:

- **Falling Mass**: \( T - W = ma \)
  where \( T \) is the tension in the cord and \( W = mg \)

- **Disk**: \( \tau = r \times T \)
  \( \tau = -RT = I \alpha = 1/2MR^2 \times a_i/R = 1/2MR a_i \)

Remember that \( a_i = a_y \)

From the torque equation: \( T = -Ma \), plug this into \( T - W = ma \):

\[
-\frac{1}{2} Ma - mg = ma
\]

and solve for acceleration

\[
a = \frac{-g}{(1 + \frac{M}{2m})}
\]

Given the measured deceleration, what is the frictional torque? Once the cord has slipped off the disk and the disk decelerates to a stop, the following expressions are relevant (see Fig. 3):

- **Disk**: \( \tau_{\text{frict}} = r \times F_{\text{frict}} \)

\[
= I \alpha = \frac{1}{2} MR^2 (\frac{a_{\text{decel}}}{R})
= \frac{1}{2} MR a_{\text{decel}}
\]

Calculate the following quantities:
1. The theoretical acceleration of the falling mass/disk
2. The frictional torque on the disk. Calculate the torque due to the hanging mass.
Before You Leave the Lab:
Study I: Make sure you have all your data tabulated and errors estimated.
Study II: Make sure you record your slope values and the errors from the fits. Print out the plot of the data, which contains the regression statistics.
Study III: Calculate the theoretical values for the acceleration and torques.

Discussion Questions:
1. Energy Conservation: Using the relevant data from your table in Study I, do you find total mechanical energy to be conserved within errors? What sources of systematic error might contribute to any differences?

2. Explain the energy of the system: where is the potential energy maximum? Where is the kinetic energy maximum? Draw pictures (this can be your lab diagram)!

3. Acceleration: Do you find that the accelerations found in Study I and Study II agree within errors? Do they agree with the calculated value? \( a = \frac{-g}{1 + \frac{M}{2m}} \)

4. How would an error in the measurement of \( R \) affect the calculation of the velocity for Study II?

5. Deceleration and Frictional Torque: Is the deceleration comparable to or much smaller than the acceleration (in magnitude)? Are we justified in ignoring friction in the bearings in Study I?