Buoyant Force and Archimedes' Principle

**Introduction:** Buoyant forces keep Supertankers from sinking and party balloons floating. An object that is more dense than a liquid will sink in that liquid. If it is less dense, it will float. Archimedes' Principle states: *A body wholly or partially immersed in a fluid will be buoyed up by a force equal to the weight of the fluid that the body displaces.*

As an equation: \( F_B = \rho_{\text{liquid}} g V_{\text{object}} \). Given \( \rho_{\text{liquid}} = \rho_{\text{liquid}} \), the buoyant force acting on a submerged object is \( F_{\text{Buoyant}} = \rho_{\text{liquid}} g V_{\text{object}} \). \( F_B \) can be found by measuring the difference between the weight of the object measured in air and the apparent weight of the submerged object.

**Study I: Buoyant Force on fully submerged objects.** You will measure \( F_B = W_{\text{in Air}} - W_{\text{in Water}} \) using an equal arm balance for several objects and plot \( F_B \) vs Volume to calculate \( \rho_{\text{water}} \). As you take your measurements consider the following question: *Why does \( F_B \) depend on \( \rho_{\text{water}} \) rather than \( \rho_{\text{object}} \)?*

**Procedure:**

1. Measure and record the diameter and thickness (neglect uncertainties) of the cylinders using the Vernier caliper. Measure the masses.

<table>
<thead>
<tr>
<th>cylinder ID</th>
<th>d, diameter (cm)</th>
<th>t, thickness (cm)</th>
<th>M, mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iron</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Copper</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Check that your equal arm balance is high enough to easily suspend the masses, but low enough to immerse the hanging masses in the container provided. See the diagram on the following page.

3. Test whether your equal arm balance is well balanced: with no additional weights, its center of mass should be close to the middle of the meter stick. Your equal arm balance needs to be perfectly balanced, otherwise you will have systematic errors. The accuracy of the measurements may not be as good as the precision. If you are careful, the systematic errors should be fairly small and they will tend to cancel when you take the difference between masses.

4. **Check for systematic errors:** Determine how much mass is required to balance a cylinder (DON'T forget the 50g mass hanger). If the cylinder/balancing mass values are very different, adjust the balance.
5. Using your balance, hang the cylinder from one hanger clamp, and the mass hanger with standard masses from the other hanger clamp as shown. Measure the mass required to balance the hanging cylinder in air and water.

![Image of balance setup](image)

*Conditions for static equilibrium: if the equal arm balance is symmetric about the knife edge, the meter stick should remain balanced when equal weights are suspended from the two hanger clamps.*

**Data Table 2:** The three cylinders can be used alone or combined.

- Make sure that the cylinders do not rest on the bottom or rub against the side of the container when making the measurements. Your water level will affect your results if you are not careful. Make sure it is high enough.

- *The sensitivity of this balance should, with some care, be better than 1 gram. Since the smallest standard mass provided is 1 gram you may be able to determine mass to a precision of perhaps half of a gram by estimation.*

<table>
<thead>
<tr>
<th>cylinder used</th>
<th>Balancing mass in air (g)</th>
<th>Balancing mass in water (g)</th>
<th>( \Delta m \pm \delta \Delta m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum (A)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iron (I)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Copper (C)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A + I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A + C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I + C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A + C + I</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. By hand or with Graphical Analysis, calculate the values of \( \Delta m \) (in grams) for each combination of cylinders in Table 2. Given an error in each mass measurement of .5 g, what is the error in \( \Delta m = m_{\text{in Air}} - m_{\text{in Water}} \)?
Analysis for Study I:

1. By hand or with GA, calculate the cylinder (and combinations*) volumes (in cm$^3$)
   \[ V_{\text{cylinder}} = \pi r^2 t \]. Remember to use radius = d/2.

   *GA won't allow you to combine different equations into a single Calculated column, so you will
   have to make a separate columns for (1) individual cylinders and (2) combinations.

2. Relate mass to $F_B$ using force diagrams: draw force diagrams for the cylinder. Write down the
   corresponding force equations. HINT: assume the tension on each string is the same when the
   meterstick is balanced.

<table>
<thead>
<tr>
<th>(1) Cylinder in Air</th>
<th>(2) Cylinder in Water</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</table>

3. Using the force equations above, relate $F_B$ to $\Delta m = m_{\text{air}} - m_{\text{water}}$: you should see that the difference is equal to $F_B/g$.

4. According to Archimedes’ principle, $F_B/g = \rho_{\text{liquid}} V_{\text{object}}$. Using Graphical Analysis, plot
   $\Delta m$ versus $V$: it should be a straight line with a slope equal to $\rho_{\text{water}}$.

5. Fit the graph with a straight line, and be sure to add the uncertainty in the fit. Compare your
   value to the accepted value for the density of water at room temperature (around 20° C).

   \[ \rho_{\text{water}} = \quad \delta \rho_{\text{water}} = \]

6. You don't need to print this graph, just make sure the slope matches (reasonably) the density of water.
Study II: Buoyant Force on partially submerged objects. Using your data from Study I, you are ready to predict the buoyant force acting on a partially submerged object. Archimedes' Principal applies to partially submerged objects, but buoyant force depends on submerged (not total) volume.

1. **Predict**: the mass difference \( \Delta m = \frac{F_b}{g} \) needed to balance a cylinder that is HALF submerged in water. **Show your calculation clearly.**

   HINT 1: Buoyant force is equal to the weight of displaced water. How much water is displaced if the cylinder is submerged half way?

   HINT 2: Should the balancing mass be more or less than that for the fully submerged object?.

   HINT 3: Should the balancing mass be more or less than that for the object in air?

2. Test your prediction using the equal arm balance. Make sure the water level isn't too high, this will affect your results.

   Were you correct?
Study III: Testing Archimedes Principal

1. Record the force meter reading.

2. Make sure the inner container is completely filled with water. Check that both buckets are empty.

3. Slowly lift the container filled with water so that the mass displaces water. Watch the force meter change reading as you do so.

   \[ F = \text{___________} \]

4. Stop when the mass is completely submerged and record the meter reading.

   \[ F = \text{___________} \]

5. **Prediction:** What will the meter read if you pour the water from the catch bucket to the upper bucket and keep the cylinder under water? **WHY?**

6. Pour the water from the catch bucket into the upper bucket and re-submerge your cylinder. Record the meter a third time. Was your prediction correct?

   \[ F = \text{___________} \]

**Discussion Questions** (can be answered outside the lab)

1. **Explain** the concept of buoyant force (use force diagrams/pictures).

2. **Study I:** Do you get agreement with the accepted density of water? The density of water depends on its temperature and that was not controlled carefully in this experiment. Is this a problem?

3. **Study I:** Why does \( F_B \) depend on \( \rho_{\text{water}} \) rather than \( \rho_{\text{object}} \)? Use diagrams/pictures. HINT: it has to do with the pressure on the object at different heights.

4. **Study II:** Did you accurately predict the amount of mass required to balance a cylinder that has been half-way submerged? Explain. Describe the physics of Study II.

5. **Study III:** Was your prediction correct? Explain. Describe the physics of Study III.