Charge to Mass Ratio of the Electron

**Purpose:** Students will observe the interaction between a magnetic field and an electron beam, and measure the charge to mass ratio of the electron.

**Historical Introduction:**

While the concept of “atom” dates back to the ancient Greeks, the current picture of an atom is only a century old. The story starts with the invention of the first working incandescent light bulb by Warren De la Rue in 1820 and subsequently improved by several other inventors. It turns out that there are many interesting phenomena associated with passing electric currents through various gasses. By 1857, German physicist Heinrich Geissler was making glass tubes containing rarified gasses and metal electrodes. The gas glowed brightly when high voltage was applied to the electrodes. Eventually, these “Geissler tubes” were made to glow in many fanciful shapes and colors. Besides being the ancestors of today’s neon sign, Geissler tubes opened the entire science of electrical discharges in gases. By 1869, better vacuum pumps were becoming available. By 1869, German physicist Johann Hittorf found that when the pressure was low enough, the gas no longer glowed but the glass at the end of the tube did. He concluded that mysterious rays, named “cathode rays” by Eugen Goldstein, were streaming from the cathode (negative electrode) of the tube. Physicists were very much puzzled by the cathode rays and other gas discharge phenomena. Sir William Crookes investigated these mysterious rays and concluded that they were particles, rather than waves. By steering the streams of particles with electric and magnetic fields, J.J. Thompson was able to measure the particles’ charge and mass. These measurements are the reason why J.J. Thompson is credited with the discovery of the electron. John Stoney had speculated about the existence of the fundamental unit of electric charge back in 1874 and even named it the “electron”, but Thompson’s experiments in 1897 convincingly showed that electrons were real.

If you are getting the impression that major scientific discoveries come from a lot of people inching forward over a long time, you are correct. While it’s common for one person to get the credit, the reality is that many people deserve it.

On with the story:

The incandescent light bulb was invented in 1820 by Warren De la Rue. It wasn’t very practical, in part because adequate vacuum pumps hadn’t yet been invented. Incandescent light bulbs contain a thin wire which is heated by passing electric current through it. The first commercially viable light bulbs were independently patented by Joseph Swan (England, 1878) and Thomas Edison (USA, 1879). But first, in 1875, Thomas Edison tried enclosing a metal electrode (now called the “plate”) into the same glass bulb as the heated filament of a prototype light bulb. He noticed that an electric current flowed from the filament to the plate when the filament was negatively charged. Edison patented this phenomenon, but did nothing else with it. This is still called the “Edison effect” today. Swan and Edison eventually joined forces and started the Edison and Swan Electric Light Company of London.

In 1904, British physicist John Ambrose Fleming realized that the Edison effect could be used to convert alternating current to direct current. This was the first “vacuum tube diode,” and it was the critical discovery that made modern electronics possible. The invention was followed by Lee De Forest’s 1906 invention of the vacuum tube triode, which amplified small electrical currents.
The triode was followed by the tetrode, pentode, and others. Except for high-power applications, these have been replaced by transistors and integrated circuits. A 1930’s vacuum tube radio sat in a cabinet in the corner of the living room, and a 1940’s computer occupied a fair-sized building. A 1960’s transistor radio could be carried around in a pocket, and a 1960’s computer could be launched on a rocket. Today, you can literally stick all that and more in your ear.

The last household vacuum tube to be replaced is the cathode-ray tube, or CRT. The CRT was the big “picture tube” of a television or computer monitor. In a CRT, an “electron gun” fires a beam of electrons at a screen. Magnetic fields focus the beam, and rapidly changing electric fields sweep it across the face of the tube. The front of the tube is coated on the inside with chemicals (called “phosphors”) that glow when the electrons hit them. By varying the intensity of the electron beam at different times, the television picture is made to appear on the tube. Which bring us to…

**Introduction to This Lab:**

In this lab, you will use a magnetic field to deflect the electron beam inside of a CRT. By doing so, you will determine the charge-to-mass ratio of the electron.

**Apparatus:**

The apparatus for this experiment will be set up for you. However, you need to understand how it works. **DO NOT TURN ANYTHING ON YET!**

<table>
<thead>
<tr>
<th>Magnet Current Power Supply</th>
<th>CRT with Mirror Scale Electron beam is blue</th>
<th>DMM for Accelerating Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leave current knob at maximum Adjust voltage knob</td>
<td>Don’t exceed 2.0 A</td>
<td>DMM for Magnet Current</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Electron Gun Filament Power Supply</th>
<th>Helmholtz Magnet Coils</th>
<th>Accelerating Voltage Power Supply</th>
<th>Don’t exceed 300 V</th>
</tr>
</thead>
</table>
How the electron gun works:
The CRT contains an “electron gun” that shoots out a beam of electrons. You can clearly see the gun through the glass tube. When the gun is turned on, its filament glows orange-hot. This heats the cathode and literally boils the electrons out of the metal. This happens in light bulbs, too. But unlike in a light bulb, we’re going to do something with them.

Our electron gun is basically a tiny particle accelerator. After the filament, the electrons are attracted toward an anode. This is an electrode placed a few millimeters from the cathode which has a hole so that the electrons can pass through it. The high voltage power supply puts a large positive voltage on the anode. This attracts the electrons and gives them a large kinetic energy, which is simply

$$K.E. = eV$$  \hspace{1cm} \text{Eq. 1}

where $e$ is the electron charge and $V$ is the accelerating voltage. The unit of kinetic energy is the eV, or electron volt. This can easily be converted to Joules by using the conversion factor $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$. The electron volt is the most commonly used energy unit in atomic, nuclear, and particle physics.

In your apparatus, the power supply under the tube provides the filament current. An external power supply, which you will adjust, provides the high voltage for accelerating the electron beam. The tube will work best between 250V and 300V. **DO NOT EXCEED 300 Volts!**

How the magnetic deflection works:
Once the electron beam is accelerated, we will steer it in circles using a uniform magnetic field. The two large black rings are a set of Helmholtz Coils. These are mounted so that their separation is equal to the radius of the coils. Recall that an electric current flowing through a wire produces a magnetic field. It turns out that this very special geometry produces a uniform magnetic field in the center of the coils. The direction of the field will be along the axis of the coils.

![Magnetic field in a pair of Helmholtz coils](image)
The magnetic field from the Helmholtz coils is given by the equation

\[ B(\text{Tesla}) = 2 \left[ \frac{\mu_0 n I R^2}{2 \left( R^2 + \frac{R^2}{4} \right)^{3/2}} \right] = \left( \frac{4}{5} \right)^{3/2} \frac{\mu_0 n I}{R} = \left( 8.99 \times 10^{-7} \frac{T \cdot m}{A} \right) n I \]

Eq. 2

where \( I \) is the current through the coil in Amperes, \( n \) is the number of turns of wire in the coils, and \( R \) is the radius of the coils in meters. In your apparatus, the radius is 0.15 m. The number of turns of wire is posted on a label on the apparatus.

Magnetic fields exert forces, called Lorentz forces, on charged particles. The situation is greatly simplified if we assume (as we can, in this apparatus) that the velocity of the particle is at right angles to the magnetic field. The force on the charged particle will then be at right angles to both the field and the velocity, as shown below:

![Force on a positively charged particle in a magnetic field.](image)

Fig. 3: Force on a positively charged particle in a magnetic field. The force on a negatively charged particle will be in the opposite direction.

This is a bit counter-intuitive. If you bang a hammer down on a nail, the nail moves in the direction of the bang. But this situation is more like banging down on an open tube of toothpaste, where the toothpaste squirts off to the side. You will see this in the experiment. Mathematically, the Lorentz force is given by

\[ \vec{F} = q \vec{v} \times \vec{B} \]

Eq. 3

If \( \vec{F}, \vec{v}, \) and \( \vec{B} \) are mutually perpendicular (as they are in this experiment), the Lorentz force steers the particle in a circle. We have

\[ F = qvB = \frac{mv^2}{r} \]

Eq. 4
where \( q \) is the particle’s charge, \( v \) its velocity, \( m \) its mass, and \( r \) the radius of its path. Note that in English, Equation 4 simply states that the magnetic Lorentz force is a centripetal force. If we combine Equation 1 with Equation 4 and substitute the electron’s charge \( e \) for the general charge \( q \), we have

\[
\frac{e}{m} = \frac{2V}{B^2r^2}
\]

Eq. 5

where \( V \) is the accelerating voltage and \( B \) is the magnetic field.

Equation 5 doesn’t let us find \( e \), and it doesn’t let us find \( m \). All we can measure in this experiment is the ratio \( e/m \). That’s still useful. Physicists use exactly this method to identify particles produced in giant particle physics experiments. Some of the magnets are as big as a house. Lightweight particles like electrons and their antimatter counterparts, the positrons, are steered quite easily by the magnet. Heavier particles like pions and protons have paths that are curved much more gently. And neutral particles like the neutron, gamma ray, and neutrino are not affected by the magnetic field at all.

**Getting Started:**

Please work in groups of three or four students for this lab.

Turn on the DMM’s and the power supplies. Allow the filament to warm up for at least 60 seconds.

While you’re waiting, record the radius and number of turns of the Helmholtz coils in the Data Sheet.

Turn the Helmholtz coil current down to zero. To adjust the Helmholtz coils, leave the “current” knob at its maximum and adjust the “voltage” knob while watching the DMM. Note that there are two sets of voltage and current knobs: coarse and fine. You’ll be working mostly with the coarse control.

**WARNING! Do NOT exceed 300V of accelerating voltage or 2.0 A of magnet current!**

After the filament has warmed up, gradually increase the accelerating voltage. You should see the electron beam start to emerge from the electron gun. The beam is blue because the electrons are colliding with trace amounts of helium gas in the tube. Every collision causes the electron to lose a bit of energy. If you are careful, you can see that the length of the electron beam increases with energy.

Next, set the accelerating voltage to approximately 275V. Gradually increase the magnet current to 1.0 V or so. You should see the electron beam bend around in a circle. Observe what happens when you gently rotate the tube in its socket. The electron beam will form a spiral. Adjust the tube so that the electron beam circles around and strikes the back of the electron gun.

If you have difficulty with this step, please call a TA for help.
You will be measuring the radius of the electron beam, using the ruler that is built into the back of the apparatus. Hold your head so that the electron beam overlaps its image in the mirrored ruler. When the beam and its image are lined up, parallax will be eliminated and you will be able to make an accurate reading.

The other measurement trick is to measure the radius of the OUTSIDE edge of the electron beam. As the electrons travel around the circle, they collide with helium atoms and lose energy. The electrons that are on the outside of the circle have lost the least energy. It may be difficult to tell just where the outside of the beam is located. Just cross your eyes and do your best.

To determine the beam radius, measure the radius on both sides of the beam circle. Add your readings and divide by two.

**Experiment Question 1: What is the charge-to-mass ratio for the electron?**

- Set the magnet current to approximately 1.0 A and set the accelerating voltage to a low value that gives you a good beam circle.
- Vary the accelerating voltage, being careful not to exceed 300 V. Measure the radius on both sides of the beam circle at several different voltages.
- EACH PERSON should measure each data point independently. Take the average of the readings.
- Estimate values for the error $\delta r$ in the measured radius. The error will be determined more by the fuzziness of the beam and the sharpness of your vision than the scale on the mirrored ruler. Use your judgment.

$$V = \left( \frac{e}{m} \right) \frac{B^2 r^2}{2}$$  \hspace{1cm}  \text{Eq. 6}

- Rearranging Equation 5 yields Equation 6. Graph $V$ vs. $r^2$, with error bars. Use the slope of the graph to determine $(e/m)$. 

Experiment Question 2: How does the beam’s radius depend on B?

- Set the magnet current to 1.0 A and set the accelerating voltage to a low value that gives you a good beam circle.
- Vary the magnet current. Measure the radius at several different currents.
- EACH PERSON should measure each data point independently. Take the average of the readings.
- Estimate values for the error $\delta r$ in the measured radius. The error will be determined more by the fuzziness of the beam and the sharpness of your vision than the scale on the mirrored ruler. Use your judgment.

$$ r = \frac{2V}{e \left(\frac{\text{m}}{\text{e}}\right)^{\frac{1}{B}}} $$  \hspace{1cm} \text{Eq. 7}$$

- Rearranging Equation 5 yields Equation 7. Graph $r$ vs. $1/B$, with error bars. Does the slope of this graph agree with what you would expect from Equation 7 using your measured value of $e/m$? Or does it agree better with the accepted value? Please discuss this in the Analysis Questions on the data sheet.

Clean Up:

- Do not disconnect the apparatus.
- Turn the accelerating voltage and magnet current down to zero, then turn everything off.
- Please dispose of trash and papers in the proper receptacles.
Data Sheet and Analysis Questions

Getting Started:

Helmholtz Coil Radius: R = ___________  Helmholtz Coil Number of Turns: n = ______

Experiment 1: What is the charge-to-mass ratio of the electron?

- This table is for YOUR data.
- Remember, EACH PERSON should measure the radii of the beam circle at each voltage.
- You should have at least six different voltages. You may do more, if you like.
- You will combine your data with that of your classmates on the next page.

What magnet current are you using? ___________

What is the magnetic field? ___________ (show your calculation in the margins.)

<table>
<thead>
<tr>
<th>V</th>
<th>Left side radius</th>
<th>Right side radius</th>
<th>Average radius</th>
<th>Error δr</th>
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</table>
Your GROUP’s data:

<table>
<thead>
<tr>
<th>V</th>
<th>Name:</th>
<th>Name:</th>
<th>Name:</th>
<th>Name:</th>
<th>Overall radius</th>
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<tr>
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<td>ave. r</td>
<td>ave. r</td>
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In addition, compare your errors \( \delta r \) for the measurements in radius. Discuss these in your group and decide on an overall measurement uncertainty.

**Experiment 1 Analysis Questions:**

1. What \( \delta r \) did you choose? \( \delta r = \) ____________
   
   How did you decide on this value?
2. Make a graph of V vs. \( r^2 \), with error bars. Use the 2% error in the DMM reading for your measurement uncertainty in V. Remove “connect the dots” and do a linear fit to your data. Calculate the value of \( \frac{e}{m} \) using the slope of your graph and Equation 6. Show your work below:

\[
\text{Answer:} \\
\frac{e}{m} \pm \delta(e/m) = \text{_________} \pm \text{_________}
\]

3. Does your answer agree with the accepted value of \( 1.76 \times 10^{11} \) C/kg?

What is the percent difference between your value and the accepted value? __________

**Experiment 2: How does the beam’s radius depend on B?**

- This time, one set of data will be sufficient.
- You should have at least six different currents. You may do more if you like.

What accelerating voltage are you using? ________________

<table>
<thead>
<tr>
<th>Current I</th>
<th>Magnetic field B</th>
<th>Left side radius</th>
<th>Right side radius</th>
<th>Average radius</th>
<th>Error ( \delta r )</th>
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Experiment 2 Analysis Questions:

4. What is the measurement uncertainty $\delta B$? ______________ Calculate this from the 2% error in reading the current in the DMM.

5. Make a graph of $r$ vs. $1/B$, with error bars. Remove “connect the dots” and do a linear fit to your data. Calculate the value of $(e/m)$ using the slope of your graph and Equation 7. Show your work below:

Answer:
$$e/m \pm \delta(e/m) = \phantom{.0} \pm \phantom{.0}$$

6. Does your answer agree with your measurement in Experiment 1?
   What is the percent difference between your two values? __________

7. Does your answer agree with the accepted value of $1.76 \times 10^{11}$ C/kg?
   What is the percent difference between your value and the accepted value? __________
8. There are several sources of error in this experiment:
   a. DMM error (assumed to be 2% of the reading) in the accelerating voltage
   b. DMM error (assumed to be 2% of the reading) in the magnet current
   c. Fuzziness of the electron beam
   d. Energy loss of the electrons due to collisions with the gas
   e. Energy loss of the electrons due to anode fields in the electron gun
   f. Crossed eyes
   g. Effects of the Earth’s magnetic field (approximately $10^{-5}$ T)

   Which of these errors is the most important source of error? ______ Explain.

9. How could you convince a critic that you don’t really need to (or maybe you do) worry about the Earth’s magnetic field in this experiment?

10. How could you figure out how much the energy loss due to gas collisions is affecting your data? (You don’t actually have to do the experiment, unless you have time and are in the mood.)

11. What are your overall thoughts and impressions about this lab?

**Graphs to turn in:** $V$ vs. $r^2$ and $r$ vs. $1/B$

**Clean Up:** Do not disconnect the apparatus. Turn the accelerating voltage and magnet current down to zero, then turn everything off. Please dispose of trash and papers in the proper receptacles.