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Decentralized Supervisory Control: Nondeterministic Transitions Versus Deterministic Moves

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Abstract— **This paper addresses the existence problem of the global supervisor for a distributed plant so that the closed loop system satisfies a global linear temporal logic specification given in a decomposable automaton. The paper shows that if there exist local supervisors for individual sub-plants of a distributed plant, then a global controller exists for decentralized supervisory control of the plant. This existence result is shown for two types of distributed plants: loosely cooperating (with deterministic events and disjoint synchronization on the common events) and synchronously communicating (with joint synchronization on the non-deterministic common events) distributed discrete event systems. For supervisory control of nondeterministic transition systems, a new notion of synchronized simulation relation is introduced to design the decentralized supervisor using synchronous product composition. An example is given to illustrate the concept of synchronized supervisory control of nondeterministic plants. This work is a new contribution in synchronization of nondeterministic events and actions in supervisory control.**

I. INTRODUCTION

Multi-agent systems control is an emerging rapidly-growing area that has attracted great academic and industrial attentions due to its ability to address new challenges in the control of complex systems composed of several interacting modules, which could not be tackle in the framework of classical control. For examples, such new challenges consist of coordination, synchronization, task sequencing, reconfigurability and adaptability of the components [1]. The low level behavior (precise trajectories) of such systems is treated using the classical time-driven techniques, while their high level behavior is modeled and controlled by discrete event systems (DES) [2]. The events are either inherently discrete or are captured by the abstraction techniques from continuous/hybrid systems [1], [3], [4], [5]). Supervisory control of discrete event systems was initiated by Ramadge and Wonhom [6], [7], [8] to have a high level logical treatment of systems subject to the order of occurrence of events. Decentralized supervisory control has been formulated in order to reduce the synthesis complexity, and to increase the implementation flexibility, by the collective actions of more than one supervisor [9]. Decentralized supervisory control problems arise naturally through the investigation of distributed transition systems [10]. Distributed transition systems are natural models for networked systems with distributed structure such as multi-agent systems that we are interested in [11]. According to the way that they coordinate their activities by synchronizing on common events, they are further classified into two main classes: loosely cooperating systems, where the global transition occurs based on local transitions with no communication of local information, and, synchronously communicating systems, where the transitions for some events are determined jointly for all local systems that synchronize on those events [12].

This paper deals with the decentralized supervisory control of the multiagent systems, modeled by distributed networked systems. Given a global specification, typically as a linear temporal logic (LTL) formula or its corresponding automaton [13], the problem of decentralized supervisory control of multi-agent systems consists of designing local supervisors for each agent so that the multi-agent system achieves the global specification. This design process could become increasingly complicated for a large number of agents with many interactions and also with a big size of LTL formula. One way to reduce the complexity is to decompose the LTL formula specification (or equivalently, its corresponding Buchi automaton) for each agent and design the local controllers, separately, so that when the agents work as a team, the global specification is satisfied by the group. Typically, decentralized supervisory control refers to the structure where supervisors are distributed into different nodes with different (and possibly overlapping) set of sensors and actuators [9], [11]. This structure reduces the complexity of design and cost of implementation of supervisory control due to cooperation of supervisors by sharing their sensors and actuators. However, it suffers from the lack of modularity for the large number of the agents in the system. To address this problem, we consider both supervisor and plant to be distributed. This paper addresses the analysis problem of decentralized supervisory control to ensure that the distributed system with the existing local supervisors, respects the decomposable language specification.

This work is inspired from [14] with further contributions in the following two directions. Firstly, assuming the existence of local supervisors, we have generalized the existence result of supervisor, from monolithic systems into loosely cooperating distributed systems to satisfy a decomposable global specification. Secondly, the results are extended to decentralized supervisory control of synchronously communicating distributed systems with synchronization on nondeterministic events. In such systems, the notion of simulation

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relation fails to design the supervisor due to ambiguity of nondeterministic events. For synchronously communicating systems we have proposed new notion of synchronized simulation relation to define nondeterministic supervisors for nondeterministic plants. This tool also provides a unified framework to compose synchronously communicating sub-modules as well as to combine global supervisor and synchronized distributed system to form a closed loop system that satisfies the global specifications.

The paper is organized as follows. Section II, at first, develops fundamental lemmas to be used in the proofs of the main results on decentralized supervisory control of loosely cooperating systems and then, represents the existence result of the decentralized supervisor for a loosely cooperating distributed plant from the existing local supervisors. To deal with nondeterministic transitions, the results of Section II, are extended to synchronously communicating systems in Section III. An example is given in Section IV, to illustrate the significance of synchronized simulation relation and to compare the decentralized supervisory control of synchronously communicating distributed systems with the loosely cooperating ones. The paper is finally concluded in Section V.

II. LOOSELY COOPERATING DISTRIBUTED SYSTEMS

A. Notations and Definitions

In this section, basic definitions are introduced. We at first define Transition systems that are simple symbolic models that can represent most of dynamic systems including control systems [13].

Definition 1: (Output Transition System) An output transition system T is a quintuple $T = (Q, E, \rightarrow, O, H)$ consisting of:

- A set of states Q ;
- A set of events E ;
- A transition relation $\longrightarrow \subseteq Q \times E \times Q$ in which $(q, e, q') \in \longrightarrow$ when $q \xrightarrow{e} q'$;
- An output set O ;
- An output function $H : Q \rightarrow O$.

Definition 2: (Simulation) Let $T_i = \left(Q_i, E_i, \underset{i}{\rightarrow}, O, H_i\right)$, i = 1, 2, be two transition systems with the same output set. A relation R is said to be a simulation relation from T_1 to T_2 (denoted by $T_1 \prec T_2$) if

1) $(q_1, q_2) \in R \Leftrightarrow H_1(q_1) = H_2(q_2)$
2) $(q_1, q_2) \in R \wedge q_1 \underset{1}{\overset{e}{\rightarrow}} q'_1 \Rightarrow \exists q_2 \underset{2}{\overset{e}{\rightarrow}} q'_2 |\, (q'_1, q'_2) \in R$

Mutual simulation relation is termed bisimultion relation.

To combine transition systems based on common events, product composition is defined as:

Definition 3: (Product Composition) Let $T_i = \left(Q_i, E_i, \underset{i}{\rightarrow}, O, H_i\right), i = 1, 2, ..., n$ be transition systems with common output set. The product composition of $T_i, i = 1, 2, ..., n$ denoted by $\overset{n}{\underset{i=1}{\times}} T_i = T_1 \times \ ... \times \ T_n$, is the product distributed transition system $\overset{n}{\underset{i=1}{\times}} T_i = \left(Q_{\overset{n}{\underset{i=1}{\times}}}, E_{\overset{n}{\underset{i=1}{\times}}}, \longrightarrow_{\overset{n}{\underset{i=1}{\times}}}, O, H_{\overset{n}{\underset{i=1}{\times}}}\right)$ defined by:

- $Q_{\overset{n}{\underset{i=1}{\times}}} = \{(q_1, ..., q_n) \in \prod_{i=1}^{n} Q_i\}$;
- $E_{\overset{n}{\underset{i=1}{\times}}} = \bigcap_{i=1}^{n} E_i$;
- $(q_1, ..., q_n) \xrightarrow[1,2,...,n]{e} (q'_1, ..., q'_n)$ if $q_i \underset{i}{\overset{e}{\rightarrow}} q'_i$ $\forall i \in \{1, ..., n\}$;
- $H_{\overset{n}{\underset{i=1}{\times}}}(q_1, ..., q_n) = H_i(q_i) \, \forall i \in \{1, ..., n\}$

B. Supervisory Control of Loosely Cooperating Systems Using Product Composition

The following lemma represents the notion of supervisory control for a monolithic system, using product composition and simulation relation.

Lemma 1: (Supervisor Realization: Lemma 4.2 in [14]) Let T_Σ be the transition system associated with the control systems $\Sigma : \dot{x} = f(x, u)$ and T_Δ be its symbolic model. If the control specifications are given by a linear temporal logic (LTL) formula represented by Buchi automaton T_S, which has the same observation space as T_Δ, an automaton $T_C \prec T_S \times \ T_\Delta$, with the same output set is a controller for T_Δ that enforces the closed loop system to satisfy the specifications, i.e.,

$$T_C \prec T_S \times \ T_\Delta \Rightarrow T_C \times \ T_\Delta \prec T_S$$

Lemma 1 tells us that defining $T_C \prec T_S \times \ T_\Delta$ causes the closed loop system $T_C \times \ T_\Delta$ does not violate the specifications, as $T_C \times \ T_\Delta \prec T_S$ then $\mathcal{L}(T_C \times \ T_\Delta) \subseteq \mathcal{L}(T_S)$ meaning that the observed behavior of the closed loop system is contained in the observed behavior of the specification. It is possible to choose the controller to be $T_C = T_S \times \ T_\Delta$ which is an automaton that its langauge is the intersection of languages of the plant and the specification languages (can be implemented by the plant while respecting the desired behavior). However, this maximal choice may cause blocking. This drawback can be overcome by selecting a non-blocking sub-transition system of $T_S \times \ T_\Delta$ [14]. The existence and computation of a controller for T_Σ satisfying T_S based on the controller for T_Δ satisfying T_S is an implication of Lemma 1 as it is given by

Lemma 2: (Theorem 4.5 in [14]) Let T_Δ be the bisimilar abstraction of T_Σ. Then a controller T_C forcing the system T_Δ to satisfy the specification T_S exists iff there exists a controller T'_C forcing system T_Σ to satisfy the specification T_S. Furthermore, we can take $T_C = T'_C$.

Remark 1: If we have only a simulation relation from T_Δ to T_Σ then the result of Lemma 2 reduces to only sufficient result. In this case, if a controller T_C forces the system T_Δ to satisfy specification T_S then there exists a controller T'_C forcing the system T_Σ to satisfy the specification T_S, and we can take $T_C = T'_C$. However, if such controller fails to exist for T_Δ then we can say nothing about the existence of controller for T_Σ fulfilling T_S.

For the rest of the paper, we consider T_Δ to be bisimilar model of T_Σ and therefore, designing a controller for T_Δ is equivalent to the synthesis of supervisor for T_Σ.

We now show that if there exist supervisors for local loosely cooperating systems, composed with product composition, deriving individual systems to satisfy local specifications, then there exists a global controller such that the global closed loop system satisfies the global specification. Following three lemmas are used for the proof the main result of this section.

Lemma 3: Let T_1, T_2 and T_3 be output transition systems with the same output set. Then

$$T_1 \times\ T_2 = T_2 \times\ T_1 \qquad (1)$$

and

$$(T_1 \times\ T_2) \times\ T_3 = T_1 \times\ (T_2 \times\ T_3) \qquad (2)$$

Properties 1 and 2 represent commutativity and associativity of output product composition, respectively and can be easily derived from Definition 3.

Lemma 4: Let T_1, T_2, T_3 and T_4 be output transition systems with the same output set. If there exist simulation relations from T_1 to T_2 and from T_3 to T_4, then, there exists a simulation transition relation from $T_1 \times\ T_3$ to $T_2 \times\ T_4$, i.e,

$$(T_1 \prec T_2) \wedge (T_3 \prec T_4) \Rightarrow (T_1 \times\ T_3) \prec (T_2 \times\ T_4)$$

Proof: See the Appendix for proof. ■

Lemma 5: Let T_1, T_2 and T_3 be any output transition systems with the same output set. Then

$$(T_1 \times\ T_2 \prec T_1) \wedge (T_1 \times\ T_2 \prec T_2)\,; \qquad (3)$$

$$T_1 \prec (T_2 \times\ T_3) \Rightarrow (T_1 \prec T_2) \wedge (T_1 \prec T_3) \qquad (4)$$

and

$$T_1 \prec (T_2 \times\ T_3) \Rightarrow (T_1 \times T_2 \prec T_3) \wedge (T_1 \times T_3 \prec T_2) \qquad (5)$$

Proof: See the Appendix for proof. ■

Remark 2: Property (3) means that any transition system simulates its product composition with other transition systems. Property (4) means that if a transition system is similar to a product composition of transition systems, it is also similar to each operand of the product composition. Property (5) is particularly used for definition of supervisor where $T_C \prec T_\Delta \times\ T_S \Rightarrow T_C \times T_\Delta \prec T_S$ as Lemma 1.

The following theorem states our main result for a product loosely cooperating system, which is indeed a distributed system composed with product composition.

Theorem 1: (Loosely Cooperating Decentralized Supervisory Control) Let the plant be a loosely cooperating system represented by $T_\Delta = \overset{n}{\underset{i=1}{\times}} T_{\Delta_i}$ with an LTL specification represented by a decomposable automaton $T_S = \overset{n}{\underset{i=1}{\times}} T_{S_i}$, where T_{S_i} are the specifications associated with the local plants T_{Δ_i}. Then, designing local supervisors as $T_{C_i} \prec T_{S_i} \times T_{\Delta_i}, i = 1, 2, \cdots, n$, results that the global supervisor $T_C \prec \overset{n}{\underset{i=1}{\times}} T_{C_i}$ steers the global closed loop system $T_C \times T_\Delta$ to satisfy the global specification T_S.

Proof: Designing $T_{C_i} \prec T_{S_i} \times T_{\Delta_i}, i = 1, 2, \cdots, n$, from Lemma 1, it follows that $T_{C_i} \times T_{\Delta_i} \prec T_{S_i}$ that due to Lemma 4, it leads to $\overset{n}{\underset{i=1}{\times}} (T_{C_i} \times T_{\Delta_i}) \prec \overset{n}{\underset{i=1}{\times}} (T_{S_i})$ and using Lemma 3 it results in $\left(\overset{n}{\underset{i=1}{\times}} (T_{C_i})\right) \times \left(\overset{n}{\underset{i=1}{\times}} (T_{\Delta_i})\right) \prec \left(\overset{n}{\underset{i=1}{\times}} (T_{S_i})\right)$. ■

Remark 3: Using simulation instead of equality in definitions of local controllers can prevent the blocking problem by choosing subtransition whose states are accessible (from the initial states) and co-accessible (reachable to the marked states). However, if the languages of local closed loop systems are conflicting it is again required to select the non-blocking global system and take $Trim$ (Accessible and Co-accessible) operator to avoid blocking in the global system. Therefore $T_C \prec \overset{n}{\underset{i=1}{\times}} T_{C_i}$ follows $T_C \times T_\Delta \prec T_S$, meaning that the global closed loop behavior satisfies the global specifications using local actuations.

III. SYNCHRONOUSLY COMMUNICATING DISTRIBUTED SYSTEMS

Loosely cooperating decentralized supervisors lack to supervise the nondeterministic events. In this section, we will introduce new tools to treat the ambiguity caused by synchronization of common nondeterministic events and then we will derive the existence of supervisory control for decentralized synchronously communicating system from the existing local supervisors.

A. Notations and Definitions

To deal with nondeterminism, we define synchronized product composition similar to loosely cooperating ones except that in the new composition, nondeterministic transitions are joint by a defined communication protocol before composing transitions. Furthermore, a new notion of synchronized simulation relation is introduced, where nondeterministic transitions are joint as a vector event, before investigation of similarity. These notions are used as essential tools to synthesize the synchronized supervisory control.

Definition 4: (Synchronized Product) Let $T_i = \left(Q_i, E_i, \underset{i}{\rightarrow}, O, H_i\right)$, $i = 1, 2, ..., n$ be transition systems with common output set. The synchronously communicating product composition, or in short synchronized product composition, of T_i, $i = 1, 2, ..., n$, denoted by $T_\otimes = \overset{n}{\underset{i=1}{\bigotimes}} T_i = T_1 \bigotimes \ldots \bigotimes T_n$, is the transition system $\overset{n}{\underset{i=1}{\bigotimes}} T_i = \left(Q_{\overset{n}{\underset{i=1}{\otimes}}}, E_{\overset{n}{\underset{i=1}{\otimes}}}, \Rrightarrow, O, H_{\overset{n}{\underset{i=1}{\otimes}}}\right)$ defined by

- $Q_{\overset{n}{\underset{i=1}{\otimes}}} = \left\{(q_1, ..., q_n) \in \prod\limits_{i=1}^{n} Q_i | H_i(q_i) = H_j(q_j)\right\}$ $\forall i, j \in \{1, ..., n\}$;
- $E_{\overset{n}{\underset{i=1}{\otimes}}} = \{\overset{n}{\underset{i=1}{\cap}} E_i\} \cup \{\prod\limits_{i=1}^{m} E_i, m \leq n\}$;
- $(q_1, ..., q_n) \overset{e}{\rightarrow} (q'_1, ..., q'_n)$ for disjoint transitions $q_i \underset{i}{\overset{e}{\longrightarrow}} q'_i$ $\forall i \in \{1, ..., n\}$ (deterministic actions);

- $\begin{bmatrix} q_1 \\ \vdots \\ q_m \end{bmatrix} \rightarrow_e \begin{bmatrix} q_1' \\ \vdots \\ q_m' \end{bmatrix}$ for joint transitions $q_i \xrightarrow[i]{e} q_i'$ $\forall i \in \{1, ..., m\}$(non-deterministic actions)
- $H_{\bigotimes_{i=1}^{n}} (q_1, ..., q_n) = H_i(q_i)\ \forall i \in \{1, ..., n\}$
- $H_{\bigotimes_{i=1}^{n}} \left(\begin{bmatrix} q_1 \\ \vdots \\ q_m \end{bmatrix} \right) = H_i(q_i)\, i \in \{1, ..., m\}$, m is the number of nondeterministic transitions.

Notion of synchronously communicating simulation relation is defined as follows and will be used for supervisory control of synchronously communicating systems.

Definition 5: (Synchronized Simulation) Let $T_i = \left(Q_i, E_i, \underset{i}{\rightarrow}, O, H_i\right)$, i = 1, 2, be two transition systems with the same output map. A relation R is said to be a synchronously communicating simulation relation, or synchronized simulation relation, from T_1 to T_2 (or T_2 simulates T_1 synchronously communicating that is denoted by $T_1 \sqsubset T_2$) if

1) $(q_1, q_2) \in R \Rightarrow H_1(q_1) = H_2(q_2)$
2) $((q_1, q_2) \in R) \wedge \left(q_1 \xrightarrow[1]{e} q_1'\right) \Rightarrow \exists q_2 \xrightarrow[2]{e} q_2' | (q_1', q_2') \in R$
3) $\left(\bar{q}_1 = \begin{bmatrix} q_{11} \\ \vdots \\ q_{1m} \end{bmatrix}, \bar{q}_2 = \begin{bmatrix} q_{21} \\ \vdots \\ q_{2m} \end{bmatrix}\right) \in R \Rightarrow$ $H_1(\bar{q}_1) = H_2(\bar{q}_2) = H_i(\bar{q}_j), i = 1, 2, j = 1, ..., m.$
4) $\left(\begin{bmatrix} q_{11} \\ \vdots \\ q_{1m} \end{bmatrix}, \begin{bmatrix} q_{21} \\ \vdots \\ q_{2m} \end{bmatrix} \right) \in R$ $\wedge \left(\begin{bmatrix} q_{11} \\ \vdots \\ q_{1m} \end{bmatrix} \underset{1}{\rightarrow}_e \begin{bmatrix} q_{11}' \\ \vdots \\ q_{1m}' \end{bmatrix} \right) \Rightarrow$ $\exists \left(\begin{bmatrix} q_{21} \\ \vdots \\ q_{2m} \end{bmatrix} \underset{2}{\rightarrow}_e \begin{bmatrix} q_{21}' \\ \vdots \\ q_{2m}' \end{bmatrix} \right) |$ $\left(\begin{bmatrix} q_{11}' \\ \vdots \\ q_{1m}' \end{bmatrix}, \begin{bmatrix} q_{21}' \\ \vdots \\ q_{2m}' \end{bmatrix} \right) \in R$

for some $m \in \{1, 2, \cdots, n\}$, where $\xrightarrow{e}$ and $\underset{e}{\longrightarrow}$ denote the disjoint and joint transitions, respectively.

Based on these definition, we can now represent the existence results of synchronized supervisory control for monolithic and decentralized systems.

B. Supervisory Control of Synchronously Communicating Systems Using Synchronized Product Composition

Following three lemmas are used for the proof of the main results of this Section.

Lemma 6: Let T_1, T_2 and T_3 be output transition systems with the same output set. Then

$$T_1 \bigotimes T_2 = T_2 \bigotimes T_1 \tag{6}$$

and

$$\left(T_1 \bigotimes T_2\right) \bigotimes T_3 = T_1 \bigotimes \left(T_2 \bigotimes T_3\right) \tag{7}$$

Properties (6) and (7) refer to the commutativity and associativity of of output synchronized product composition, respectively, and are directly derived from definition of synchronized product composition (Definition 4). The only difference of this lemma with Lemma 3 is that in the case of synchronized product, the states can be either scalar (disjoint) or vector (joint before transitions).

Lemma 7: Let T_1, T_2, T_3 and T_4 be output transition systems with the same output set. If there exist synchronized simulation transition relations from T_1 to T_2 and from T_3 to T_4, then, there exists a synchronized simulation transition relation from $T_1 \bigotimes T_3$ to $T_2 \bigotimes T_4$, i.e.,

$$(T_1 \sqsubset T_2) \wedge (T_3 \sqsubset T_4) \Rightarrow (T_1 \bigotimes T_3) \sqsubset (T_2 \bigotimes T_4)$$

Proof: See the Appendix for proof. ■

Lemma 8: Let T_1, T_2 and T_3 be any output transition systems with the same output set. Then

$$\left(T_1 \bigotimes T_2 \sqsubset T_1\right) \wedge \left(T_1 \bigotimes T_2 \sqsubset T_2\right); \tag{8}$$

$$T_1 \sqsubset (T_2 \bigotimes T_3) \Rightarrow (T_1 \sqsubset T_2) \wedge (T_1 \sqsubset T_3) \tag{9}$$

and

$$T_1 \sqsubset (T_2 \bigotimes T_3) \Rightarrow \left(T_1 \bigotimes T_2 \sqsubset T_3\right) \wedge \left(T_1 \bigotimes T_3 \sqsubset T_2\right) \tag{10}$$

Proof: See the Appendix for proof. ■

Remark 4: Property (8) means that any transition system synchronously simulates its synchronized product composition with other transition systems. Property (9) means that if a transition system is synchronously similar to a synchronized product composition of transition systems, it is also synchronously similar to each operand of synchronized product composition. The property (10) is particularly used for definition of supervisor where $T_C \sqsubset T_\Delta \bigotimes T_S \Rightarrow T_C \bigotimes T_\Delta \sqsubset T_S$ as Theorem 2.

The main result for monolithic supervisory control of synchronously communicating system is represented as the following theorem.

Theorem 2: (Synchronized Product Supervisory Control) Let T_Δ and T_S be automata corresponding to the plant dynamic and the specification represented in LTL formula with the same output map. Then, a controller $T_C \sqsubset T_\Delta \bigotimes T_S$ reinforces the closed loop system to respect the specification T_S, i.e., $(T_C \sqsubset T_\Delta \bigotimes T_S) \Rightarrow (T_C \bigotimes T_\Delta \sqsubset T_S)$.

Proof: This theorem is a consequent of Lemma 8. ■

We now, show that if there exist supervisors for local synchronously communicating systems deriving individual systems to satisfy local specifications, then, there exists a global controller such that the global closed loop system satisfies the global specification.

Theorem 3: Let the plant be a synchronously communicating system, represented by $T_\Delta = \bigotimes_{i=1}^{n} T_{\Delta_i}$, with an LTL specification represented by a decomposable automaton

$T_S = \bigotimes_{i=1}^{n} T_{S_i}$, where T_{S_i} are the specification associated with the local plants T_{Δ_i}. Then, designing local supervisors as $T_{C_i} \sqsubset T_{S_i} \bigotimes T_{\Delta_i}, i = 1, 2, \cdots, n$, results that the global supervisor $T_C \sqsubset \bigotimes_{i=1}^{n} T_{C_i}$ derives the global closed loop system $T_C \bigotimes T_\Delta$ to satisfy the global specification T_S.

Proof: Designing $T_{C_i} \sqsubset T_{S_i} \bigotimes T_{\Delta_i}, i = 1, 2, \cdots, n$, from Theorem 2, it follows that $T_{C_i} \bigotimes T_{\Delta_i} \sqsubset T_{S_i}$ that due to Lemma 7, it leads to $\bigotimes_{i=1}^{n} (T_{C_i} \bigotimes T_{\Delta_i}) \sqsubset \bigotimes_{i=1}^{n} (T_{S_i})$ and using Lemma 6 it results that $\left(\bigotimes_{i=1}^{n} (T_{C_i})\right) \bigotimes \left(\bigotimes_{i=1}^{n} (T_{\Delta_i})\right) \sqsubset \left(\bigotimes_{i=1}^{n} (T_{S_i})\right)$, meaning that the global closed loop behavior satisfies the global specifications using local actuations. ∎

IV. EXAMPLES

An example of synchronously communicating distributed systems is shown in Figure 1, where two subsystems are composed using synchronized product to form the overall system $T_\Delta = T_{\Delta_1} \bigotimes T_{\Delta_2}$ with joint transitions defined on non-deterministic event c and disjoint transitions defined on deterministic events a and b, as the following protocol.

- $\begin{bmatrix} q_1 \\ q_1' \end{bmatrix} \rightarrow_c \begin{bmatrix} q_2 \\ q_2' \end{bmatrix}, \begin{bmatrix} q_1 \\ q_1' \end{bmatrix} \rightarrow_c \begin{bmatrix} q_4 \\ q_4' \end{bmatrix}$
- $q_2 \xrightarrow[1]{a} q_3,\ q_3 \xrightarrow[1]{b} q_1,\ q_4 \xrightarrow[1]{b} q_5,\ q_5 \xrightarrow[1]{a} q_1$
- $q_2' \xrightarrow[2]{a} q_3',\ q_3' \xrightarrow[2]{b} q_1',\ q_4' \xrightarrow[2]{b} q_5',\ q_5' \xrightarrow[2]{a} q_1'$,

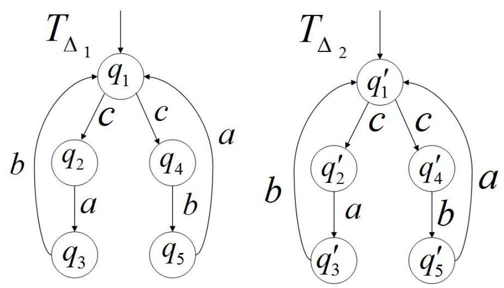

Fig. 1. Example of synchronously communicating distributed system.

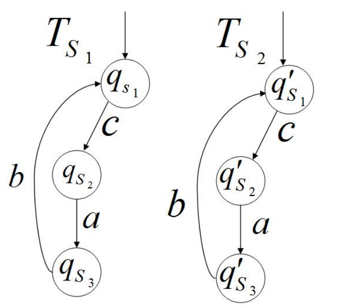

Fig. 2. Example of distributed supervisor.

By this protocol, synchronized product of two systems, demands them to have the same order of events a and b, while the distributed specification $T_S = T_{S_1} \bigotimes T_{S_2}$ (See Figure 2) accepts only the ab order. Because of nondeterminism, a synchronized product is used to realize the supervisory control of the system. The non-blocking decentralized controller $T_C = T_S \bigotimes T_\Delta$ is synchronously bisimilar (mutually similar) to T_S, driving the closed loop system to be bisimilar and hence language equivalent to the specification. Now, consider Figure 3, where the plant is a loosely cooperating distributed system that is defined as product composition of T_{Δ_1} and T_{Δ_2}. All events of this system are deterministic and hence, product composition of local supervisors shown in Figure 2 is sufficient in order to drive the closed loop system to the aforementioned specification.

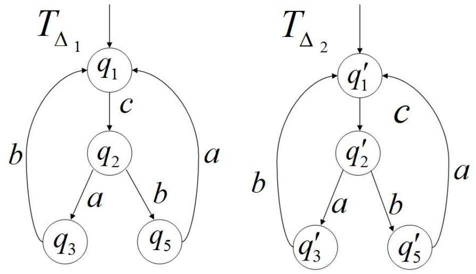

Fig. 3. Example of loosely cooperating distributed system.

V. CONCLUSION

In this paper, we examined the existence of a global supervisor for loosely cooperating as well as synchronously communicating distributed systems from the existence of the local supervisors. The paper has two main contributions: Firstly, the supervisory control approach for a monolithic system based on simulation relation and product composition, is generalized for a decentralized loosely cooperating system. Secondly, To deal with nondeterminism, a new notion of synchronized simulation relation is introduced and then decentralized supervisory control of synchronously communicating systems is proposed using synchronized simulation relation and synchronized product composition.

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VI. APPENDIX

A. Proof for Lemma 4

Proof: From definition of product composition of $T_1 \times T_3$, for all $(q_1, q_3) \in Q_1 \times \ Q_3$ we have

$$q_1 \longrightarrow_1 q_1' \tag{11}$$

$$q_3 \longrightarrow_3 q_3' \tag{12}$$

$$H_1(q_1) = H_3(q_3) = H_{T_1 \times \ T_3}(q_1, q_3) \tag{13}$$

Since $T_1 \prec T_2$ and $T_3 \prec T_4$, from (11), (12) and definitions of product composition and simulation relation, it follows, respectively

$$\left\{ \begin{array}{l} \exists q_2' \in Q_2, q_2 \longrightarrow_2 q_2' \\ H_1(q_1) = H_2(q_2) = H_{T_1 \times \ T_3}(q_1, q_3) = H_{T_2 \times \ T_4}(q_2, q_4) \end{array} \right. \tag{14}$$

$$\left\{ \begin{array}{l} \exists q_4' \in Q_4, q_4 \longrightarrow_4 q_4' \\ H_3(q_3) = H_4(q_4) = H_{T_1 \times \ T_3}(q_1, \ q_3) = H_{T_2 \times \ T_4}(q_2, q_4) \end{array} \right. \tag{15}$$

From (14), (15) and (13) it implies that

$$\left\{ \begin{array}{l} \forall (q_1, q_3) \longrightarrow_{T_1 \times \ T_3} (q_1', \ q_3') \exists (q_2, q_4) \longrightarrow_{T_2 \times \ T_4} (q_2', q_4') \\ H_{T_1 \times \ T_3(q_1, \ q_3)} = H_{T_2 \times \ T_4}(q_2, q_4) \end{array} \right. \tag{16}$$

which means $(T_1 \times \ T_3) \prec (T_2 \times \ T_4)$ ■

B. Proof for Lemma 5

Proof: The proof of (3) follows from the third item in Definition 3. The proof of (4) is derived from (3) as $(T_2 \times \ T_3 \prec T_2) \wedge (T_2 \times \ T_3 \prec T_3)$ and finally, the last property, (5), can be derived from the combination of (3) and (4). ■

C. Proof for Lemma 7

Proof: From definition of synchronized product composition of $T_1 \bigotimes \ T_3$ for all disjoint states $(q_1, q_3) \in Q_1 \times \ Q_3$ we have

$$q_1 \longrightarrow_1 q_1' \tag{17}$$

$$q_3 \longrightarrow_3 q_3' \tag{18}$$

$$H_1(q_1) = H_3(q_3) = H_{T_1 \bigotimes T_3}(q_1, q_3) \tag{19}$$

Since $T_1 \sqsubset T_2$ and $T_3 \sqsubset T_4$, from (17), (18) and definitions of synchronized product composition and synchronized simulation relation it follows, respectively

$$\left\{ \begin{array}{l} \exists q_2' \in Q_2, q_2 \longrightarrow_2 q_2' \\ H_1(q_1) = H_2(q_2) = H_{T_1 \bigotimes T_3}(q_1, q_3) = H_{T_2 \bigotimes T_4}(q_2, q_4) \end{array} \right. \tag{20}$$

$$\left\{ \begin{array}{l} \exists q_4' \in Q_4, q_4 \longrightarrow_4 q_4' \\ H_3(q_3) = H_4(q_4) = H_{T_1 \bigotimes T_3}(q_1, \ q_3) = H_{T_2 \bigotimes T_4}(q_2, q_4) \end{array} \right. \tag{21}$$

From (20), (21) and (19) it implies that

$$\left\{ \begin{array}{l} \forall (q_1, q_3) \longrightarrow_{T_1 \bigotimes T_3} (q_1', \ q_3') \exists (q_2, q_4) \longrightarrow_{T_2 \bigotimes T_4} (q_2', q_4') \\ H_{T_1 \bigotimes T_3}(q_1, \ q_3) = H_{T_2 \bigotimes T_4}(q_2, q_4) \end{array} \right. \tag{22}$$

For joint states $\bar{q}_1 = \begin{bmatrix} q_{11} \\ \vdots \\ q_{1m} \end{bmatrix}$ and $\bar{q}_3 = \begin{bmatrix} q_{31} \\ \vdots \\ q_{3m} \end{bmatrix}$ we have

$$\bar{q}_1 \rightarrow_e \bar{q}_1{}' \tag{23}$$

$$\bar{q}_3 \rightarrow_e \bar{q}_3{}' \tag{24}$$

$$H_1(\bar{q}_1) = H_3(\bar{q}_3) = H_{T_1 \bigotimes \ T_3}(\bar{q}_1, \bar{q}_3) \tag{25}$$

where $\bar{q}_1{}' = \begin{bmatrix} q_{11}' \\ \vdots \\ q_{1m}' \end{bmatrix}$ and $\bar{q}_3{}' = \begin{bmatrix} q_{31}' \\ \vdots \\ q_{3m}' \end{bmatrix}$.

Again, since $T_1 \sqsubset T_2$ and $T_3 \sqsubset T_4$, from (23), (24) and definitions of synchronized product composition and synchronized simulation relation, it follows, respectively

$$\left\{ \begin{array}{l} \exists \bar{q}_2' \in Q_2, \bar{q}_2 \rightarrow_2 \bar{q}_2' \\ H_1(\bar{q}_1) = H_2(\bar{q}_2) = H_{T_1 \bigotimes T_3}(\bar{q}_1, \bar{q}_3) = H_{T_2 \bigotimes T_4}(\bar{q}_2, \bar{q}_4) \end{array} \right. \tag{26}$$

$$\left\{ \begin{array}{l} \exists \bar{q}_4' \in Q_4, \bar{q}_4 \rightarrow_4 \bar{q}_4' \\ H_3(\bar{q}_3) = H_4(\bar{q}_4) = H_{T_1 \bigotimes T_3}(\bar{q}_1, \ \bar{q}_3) = H_{T_2 \bigotimes T_4}(\bar{q}_2, \bar{q}_4) \end{array} \right. \tag{27}$$

From (26), (27) and (25), it implies that

$$\left\{ \begin{array}{l} \forall (\bar{q}_1, \bar{q}_3) \rightarrow_{T_1 \bigotimes T_3} (\bar{q}_1', \ \bar{q}_3') \exists (\bar{q}_2, \bar{q}_4) \rightarrow_{T_2 \bigotimes T_4} (\bar{q}_2', \bar{q}_4') \\ H_{T_1 \bigotimes \ T_3}(\bar{q}_1, \ \bar{q}_3) = H_{T_2 \bigotimes T_4}(\bar{q}_2, \bar{q}_4) \end{array} \right. \tag{28}$$

and the proof follows from (22) and (28), where collectively mean that $T_1 \bigotimes \ T_3 \sqsubset T_2 \bigotimes \ T_4$. ■

D. Proof for Lemma 8

Proof: The proof of (8) follows from Definitions 5 and 4. The proof of (9) is derived from (8) and transitivity of synchronized simulation. The last property, (10), can be proven using (8) and (9). ■