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Necessary and sufficient bit rate conditions to stabilize quantized Markov jump linear systems

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Abstract— **This paper studies the mean square stability of a quantized Markov jump linear system (MJLS) that closes its feedback path closed over digital networks. It investigates the effects of the finite constant feedback bit rate on the MJLS' stability. A lower bound is placed on all constant bit rates which could stabilize the MJLS. It also provides a sufficient bit rate for the MJLS, which is shown to be enough to stabilize the system through constructing an appropriate quantization policy. So it is possible to stabilize a quantized MJLS at a finite bit rate. Moreover, it can determine a bit rate range, within which the minimum bit rate required for stability lies. An example is used to verify the achieved theoretical results.**

I. Introduction

Markov jump linear systems (MJLS) are often encountered in telecommunication, manufacturing and transportation, whose parameters may be abruptly changed according to a Markov chain [1]. In last few decades, there has been much research on MJLS [2] [3].

Most results on MJLS are built upon the perfect feedback assumption, i.e., the transmitted feedback signal is accurately received. The assumption, however, may be violated when the feedback path is closed over digital communication networks [4]. Due to the digital nature of such networks, all data must be quantized before transmission, which will incur feedback information error, i.e., quantization error. Then a question arises, *will quantization error destroy the established properties of MJLS?* As the most important property of MJLS, stability is the first to check under perturbation of quantization error. Here we are concerned with mean square stability (MSS) [2]. Quantization error is determined by the feedback network's bit rate (the higher bit rate, the smaller quantization error). This paper establishes two bit rate conditions required for MSS of a MJLS, including

- **Necessary bit rate condition:** For a given MJLS, we find a lower bound on all constant bit rates which can guarantee MSS.
- **Sufficient bit rate condition:** A constant bit rate determined by the parameters of the given MJLS is shown to be able to stabilize that MJLS through constructing appropriate quantization policies.

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With the above two conditions, we can determine a bit rate range, within which the minimum bit rate to stabilize the MJLS lie.

Now we briefly review the relevant literature. The quantization literature mainly focuses on the single dynamical system case, i.e., there is no parameter jumping. The available quantization policies can be categorized into two groups, static one and dynamic one [5]. *Static quantization policies* take a constant quantization range, map each bit to a specific subset of that range in a fixed (static) manner. The attraction of static policies is the simplicity of their coding/decoding schemes[6] [7]. Their main drawback is that only the ultimate boundedness of the state, instead of asymptotic stability, can be guaranteed at a finite bit rate [8][9][10][11]. Compared with static policies, *dynamic quantization policies* may choose a time-varying quantization range and their mapping between the quantization bits and the subsets of the quantization range can also be time-varying. Although more complicated, the dynamic policies can asymptotically stabilize noise-free linear systems at a finite bit rate [12] [13] . The minimum bit rate to maintain asymptotic stability is given in [14] [15]. Due to their efficiency, dynamic quantization polices are chosen in the present paper.

There are some results on stability of quantized Markov jump linear systems. In [16] [17], static logarithmic quantization policies are constructed to stabilize the MJLS in the mean square sense. These policies, however, require an infinite bit rate (or an infinite number of bits per sample). In [18], a finite bit rate is needed to stabilize a scalar MJLS in the moment sense. Moreover, the minimum stabilizing bit rate is derived in [18]. Some efforts were made in [19] to extend the results in [18] to the more general multi-dimensional case. Due to the abrupt parameter switching, the dynamics of a multi-dimensional MJLS can be much more complicated than that of a scalar MJLS and the necessary and sufficient stability condition in [19] is flawed, which motivated the research in the present paper.

The rest of this paper is organized as follows. Section II presents the mathematical model of the quantized MJLS and some assumptions. Section III derives a lower bound on all bit rates being able to stabilize a MJLS. Section IV finds a bit rate for a MJLS and proves that rate is enough for stability through constructing a dynamic quantizer. Section V concludes this paper with some final remarks.

II. Mathematical models

This paper focuses on the system in Fig. 1.

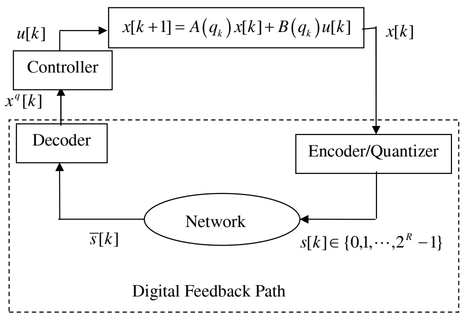

Fig. 1. A quantized Markov Jump Linear System

As shown in Fig. 1, the control plant is a MJLS. Both the system matrix $A(q[k]) \in R^{n\times n}$ and the input $B(q[k]) \in R^{n\times m}$ are driven by the mode sequence $\{q[k]\}$. Here $\{q[k]\}$ is governed by an irreducible homogeneous N-state ($q[k] \in \{q_1, q_2, \cdots, q_N\}$) Markov chain with the probability matrix $T = (t_{ij})_{N\times N}$ ($t_{ij} = P(q[k+1] = q_j|q[k] = i)$). $x[k] \in R^n$ is the state at time instant k ($= 0, 1, 2, \cdots$). $x[k]$ is quantized into one of 2^R symbols, $s[k]$, and sent over the digital communication network. Note that the fixed length coding is employed here due to its bandwidth efficiency. It is assumed that the transmitted symbol $s[k]$ is reliably received with 1-step delay, i.e., $\overline{s}[k] = s[k-1]$. The decoder uses all received symbols $\{\overline{s}[k], \overline{s}[k-1], \cdots, \overline{s}[1]\}$ to estimate the state $x[k]$. The state estimate is denoted as $\hat{x}[k]$, which can also be viewed as a quantized version of $x[k]$ with the quantization error

$$e[k] = x[k] - \hat{x}[k]. \tag{1}$$

The control input $u[k] \in R^m$ is then constructed from $\hat{x}[k]$ according to a mode-dependent static feedback law $u[k] = K(q[k])\hat{x}[k]$. For reference convenience, the system in Fig. 1 is mathematically described as

$$\left\{ \begin{array}{lcl} x[k+1] & = & A(q[k])x[k] + B(q[k])u[k] \\ u[k] & = & K(q[k])\hat{x}[k] \end{array} \right. . \tag{2}$$

It is assumed that the un-quantized system, i.e., eq. 2 with $\hat{x}[k] = x[k]$ ($e[k] = 0$), is mean square stable (MSS) under the gains $K(q[k])$, which is expressed as [2]

$$\lim_{k\to\infty} \|x[k]\|_2^2 = 0, \forall x[0] \in R^n, \tag{3}$$

where $\|\cdot\|_2$ denotes the Euclidean norm of a vector. It is also assumed that the mode sequence $\{q[k]\}$ is known by both the encoder and the decoder. Our problem is to check whether eq. 3 still holds under the non-zero quantization error $\{e[k]\}$.

For notational simplicity, $A(q[k])$, $B(q[k])$ and $K(q[k])$ are respectively denoted as A_i, B_i and K_i when $q[k] = q_i$ ($i = 1, \cdots, N$).

III. A NECESSARY BIT RATE CONDITION TO STABILIZE A QUANTIZED MJLS

In order to establish the necessary bit rate condition, we need the concept of uncertainty set [12] [14]. At time k, the controller/decoder cannot know the exact value of the state $x[k]$. It just knows that $x[k]$ lies within an uncertainty set $P[k]$, which is comprised of all possible states generated from all initial states $x[0] \in P[0]$ and the mode sequence $\{q[0], \cdots, q[k-1]\}$, i.e.,

$$P[k] = \{z|z = x[k], x[0] \in P[0], \{q[0], \cdots q[k-1]\}\}.$$

Define the volume of $P[k]$ as

$$vol(P[k]) = \int_{x\in P[k]} 1dx.$$

Now we study the evolution of $vol(P[k])$ with the transmission of $s[k]$ ($\in S = \{0, 1, \cdots, 2^R - 1\}$). $s[k]$ is generated by partitioning $P[k]$ into 2^R disjoint subsets denoted as $P_0[k], P_1[k], \cdots, P_{2^R-1}[k]$, and mapping each subset to one symbol among S. With the mapping policy and $s[k]$, the controller knows which subset $x[k]$ lies in, to say $x[k] \in P_j[k]$. So

$$\begin{aligned} & vol(P[k+1]) \\ = \; & vol\left(\{z|z = A(q[k])x[k] + B(q[k])u[k],\right. \\ & \left. x[k] \in P_j[k]\}\right) \\ = \; & vol\left(\{z|z = A(q[k])x[k], x[k] \in P_j[k]\}\right) \\ = \; & |det(A(q[k]))|vol\left(\{z|z = x[k], x[k] \in P_j[k]\}\right) \\ = \; & |det(A(q[k]))|vol(P_j[k]). \end{aligned} \tag{4}$$

Because $vol(P[k]) = \sum_{i=0}^{2^R-1} vol(P_i[k])$, we know $\max_i vol(P_i[k]) \geq \frac{1}{2^R} vol(P[k])$. $P_j[k]$ can be the subset with the largest volume, which is combined with eq. 4 to yield

$$vol(P[k+1]) \geq \frac{|det(A(q[k]))|}{2^R} vol(P[k]). \tag{5}$$

Define $V[k] = \sqrt[n]{vol(P[k])}$. Then eq. 5 can be expressed in terms of $V[k]$ as

$$V[k+1] \geq \frac{c(q[k])}{2^{R/n}} V[k], \tag{6}$$

where

$$c(q[k]) = \sqrt[n]{|det(A(q[k]))|}. \tag{7}$$

According to the definition of $V[k]$, we can place an upper bound on $V[k]$ in Lemma 3.1, whose proof is simple and omitted here.

Lemma 3.1:

$$V[k] \leq 2 \max_{x'[k]\in P[k]} \|x'[k]\|_2. \tag{8}$$

When the quantized MJLS in eq. 2 is mean square stable, $\lim_{k\to\infty} \mathbf{E}\left[\|x[k]\|_2^2\right] = 0$ for $\forall x[0] \in P[0]$. Considering eq. 8, we get

$$\lim_{k\to\infty} \mathbf{E}\left[(V[k])^2\right] = 0. \tag{9}$$

Because $V[k]$ is nonnegative and satisfies eq. 6, eq. 9 implies the following scalar MJLS is mean square stable

$$V'[k+1] = c(q[k])V'[k], \tag{10}$$

where $c(q[k])$ is defined in eq. 7. By [20], the mean square stability of the system in eq. 10 is equivalent to the stability of the following matrix S_1, i.e., all eigenvalues of S_1 lie within the unit circle.

$$S_1 = \frac{1}{2^{2R/n}} T' \times diag([c_1^2, c_2^2, \cdots, c_N^2]), \tag{11}$$

where T' is the transposition of T and $c_i = c(q_i)$ ($i = 1, \cdots, N$).

Based on the above argument, we can place a lower bound on R, which is necessary for stabilizing eq. 2.

Theorem 3.2: The quantized MJLS in eq. 2 is mean square stable only if

$$R > \frac{n}{2} log_2 \left(\lambda_{max}(\underline{S}) \right), \tag{12}$$

where $\lambda_{max}(\cdot)$ denotes the maximum eigenvalue magnitude of a matrix, $\underline{S} = T' \times diag\left(\left[\sqrt[n]{|det(A_1)|^2}, \sqrt[n]{|det(A_2)|^2}, \cdots, \sqrt[n]{|det(A_N)|^2} \right] \right)$.

Remark: Due to the constant bit rate constraint, R has to be an integer. So eq. 12 is equivalent to

$$R \geq \left\lceil \frac{n}{2} log_2 \left(\lambda_{max}(\underline{S}) \right) \right\rceil, \tag{13}$$

where $\lceil \cdot \rceil$ stands for the ceiling operation over a real number.

Remark: The above procedure can be repeated for the r-th moment stability. Without proof, we write down the necessary bit rate condition that a r-th moment stable quantized MJLS has to satisfy.

$$R > \frac{n}{r} log_2 \left(\lambda_{max}(\underline{S}_r) \right), \tag{14}$$

where

$$\underline{S}_r = T' \times diag\left(\left[\sqrt[n]{|det(A_1)|^r}, \sqrt[n]{|det(A_2)|^r}, \cdots, \sqrt[n]{|det(A_N)|^r} \right] \right).$$

The result in eq. 14 agrees with that of [18] in the scalar quantized MJLS case.

IV. A SUFFICIENT BIT RATE CONDITION TO STABILIZE A QUANTIZED MJLS

The last section provides a lower bound on constant bit rates to stabilize a MJLS. This section will give a bit rate and show it is enough to stabilize the MJLS. That result is presented in the following Theorem 4.1 and the rest of Subsection IV-A is dedicated to its proof. An example is included in Subsection IV-B to verify the theoretical results.

A. A bit rate to stabilize the quantized MJLS

Theorem 4.1: The quantized MJLS in eq. 2 can be stabilized in the mean square sense if

$$R \geq nR_0, \tag{15}$$

where

$$R_0 = \left\lceil log_2 \left(\frac{1}{2} \lambda_{max}(\overline{S}) \right) \right\rceil, \tag{16}$$

where $\overline{S} = (T' \otimes I_{n^2}) \times diag([|A_1| \otimes |A_1|, |A_2| \otimes |A_2|, \cdots, |A_N| \otimes |A_N|])$ with $|A_i|$ standing for the element-wise absolute value of A_i ($i = 1, \cdots, N$) and $\otimes$ the Kronecker product [21].

By Theorem 4.1, we know it is possible to stabilize a quantized MJLS at a *finite* bit rate. Furthermore, we can determine a range, within which the minimum stabilizing bit rate lies.

Corollary 4.2: Denote by R_{min} the minimum constant bit rate to stabilize the MJLS in eq. 2. R_{min} is bounded as

$$\left\lceil \frac{n}{2} log_2 \left(\lambda_{max}(\underline{S}) \right) \right\rceil \leq R_{min} \leq n \left\lceil log_2 \left(\frac{1}{2} \lambda_{max}(\overline{S}) \right) \right\rceil,$$

where $\underline{S}$ and $\overline{S}$ are defined as eq. 12 and 15.

In order to prove Theorem 4.1, we first substitute eq. 1 into eq. 2 to get

$$\begin{aligned} x[k+1] &= \left(A(q[k]) + K(q[k]) B(q[k]) \right) x[k] \\ &\quad + B(q[k]) e[k]. \end{aligned} \tag{17}$$

By the assumption in Section II, we know the MJLS $z[k+1] = \left(A(q[k]) + K(q[k]) B(q[k]) \right) z[k]$ is mean square stable. So the MJLS in eq. 17 (eq. 2) is also mean square stable if

$$\lim_{k \to \infty} \|e[k]\|_2^2 = 0. \tag{18}$$

Next we construct a quantizer (encoder and decoder) to guarantee eq. 18. In Section III, we defined an uncertainty set $P[k]$, which comprises of all possible $x[k]$ at time k. $P[k]$, however, may not have a good shape. We can over-bound $P[k]$ with a rectangle $U[k]$, which is characterized by its center $z^U[k]$ and its side length vector $L[k] = [L_1[k], L_2[k], \cdots, L_N[k]]^T$. A rectangle with the center of the origin and the side length L is denoted as $rect(L)$. Because $x[k] \in P[k]$ and $P[k] \subseteq U[k]$,

$$x[k] \in U[k] = z^U[k] + rect(L[k]).$$

We estimate $x[k]$ with the center $z^U[k]$ of $U[k]$, i.e., $\hat{x}[k] = z^U[k]$. So $U[k]$ can be expressed as

$$U[k] = \hat{x}[k] + rect(L[k]).$$

The quantization error $e[k] (= x[k] - \hat{x}[k])$ is bounded as

$$|e_i[k]| \leq 0.5 L_i[k]. \tag{19}$$

So the mean square convergence of $\{e[k]\}$ is implied by that of $\{L[k]\}$. The following quantizer can guarantee $\lim_{k \to \infty} \|L[k]\|_2^2 = 0$.

Partition all n sides of $U[k]$ into 2^{R_0} equal parts, which can be indexed by R_0-bit symbols s_i($i = 1, \cdots, n; s_i = 0, \cdots, 2^{R_0} - 1$). All n symbols comprises a symbol vector $s = [s_1, s_2, \cdots, s_n]$, which is actually a R-bit ($R = nR_0$) symbol. After partitioning all sides, we get a modified side length vector

$$\hat{L}[k] = \frac{1}{2^{R_0}} L[k]. \tag{20}$$

Now the original set $U[k] = \hat{x}[k] + rect(L[k])$ is partitioned into 2^{nR_0} smaller subsets $U_s[k]$,

$$U_s[k] = \hat{x}_s[k] + rect(\hat{L}[k]),$$

where $\hat{x}_s[k] = \hat{x}[k] + [x_{s_1}[k], x_{s_2}[k], \cdots, x_{s_n}[k]]^T$ with $x_{s_i}[k] = \frac{-2^{R_0} + (2s_i + 1)}{2^{R_0 + 1}} L_i[k]$ ($i = 1, \cdots, n$). Because $U[k]$

is comprised of these 2^{nR_0} smaller subsets and $x[k] \in U[k]$, there must exist s_0 such that $x[k] \in U_{s_0}[k]$. Set $s[k] = s_0$, code $s[k]$ into nR_0 bits and send these bits to the decoder through the network. Due to the reliable network transmission assumption, the decoder must receive $s[k]$ (with 1 step delay). So the encoder and the decoder agree upon $x[k] \in \hat{x}_{s[k]}[k] + rect(\hat{L}[k])$. Based on the system equation 2, the encoder and decoder update the state set $U[k+1]$, in which $x[k+1]$ lies, as

$$\begin{cases} L[k+1] = |A(q[k])|\hat{L}[k] \\ \hat{x}[k+1] = A(q[k])\hat{x}_{s[k]}[k] + Bu[k] \end{cases}, \quad (21)$$

where $|A(q[k])|$ stands for the element-wise absolute value of $A(q[k])$ and the control variable is computed as

$$u[k] = K(q[k])\hat{x}[k]. \quad (22)$$

The above quantization policy is summarized into the following algorithm.

Algorithm 1: **Quantization algorithm:**

Encoder/Decoder initialization:

Initialize $\hat{x}[0]$ and $L[0]$ so that $x[0] \in \hat{x}[0] + rect(L[0])$ and set $k = 0$.

Encoder Algorithm:

1) **Quantize** the state $x[k]$ by setting $s[k] = s$ if $x[k] \in \hat{x}_s[k] + rect(\hat{L}[k])$.
2) **Transmit** the quantized symbol $s[k]$.
3) **Update** $\hat{z}[k+1]$ and $L[k+1]$ by eq. 21 immediately before time $k+1$. Update time index, $k = k+1$ and return to step 1.

Decoder Algorithm:

1) **Compute** the control variable for time k by eq. 22.
2) **Wait** for the quantized data, $s[k]$, from the encoder.
3) **Update** $\hat{z}[k+1]$ and $L[k+1]$ by eq. 21 immediately before time $k+1$. Update time index, $k = k+1$ and return to step 1.

It can be shown that there is no state overflow, which is presented as

Proposition 4.3: Under Algorithm 1,

$$x[k] \in U[k] = \hat{x}[k] + rect(L[k]), \forall k \geq 0. \quad (23)$$

Combining eq. 20 and 21, we know

$$L[k+1] = \frac{1}{2^{R_0}}|A(q[k])|L[k]. \quad (24)$$

The above equation is a standard MJLS. By [20], we get

Proposition 4.4: Under the updating rule in eq. 24, $\lim k \to \infty \|L[k]\|_2^2 = 0$ if and only if

$$R_0 > log2\left(\frac{1}{2}\lambda_{max}(\overline{S})\right), \quad (25)$$

where $\overline{S}$ is defined as eq. 16.

By eq. 17 and 19, we know $\lim k \to \infty \|L[k]\|_2^2 = 0$ implies the mean square stability of the original system in eq. 2. So eq. 15 is enough for stabilizing the MJLS in the mean square sense. $\diamondsuit$

Remark: Algorithm 1 requires that the initial state $x[0]$ lies within a known set $U[0]$. The mean square stability, however, requires that the initial state $x[0]$ can be any vector in R^n [2]. It is possible that $x[0] \notin U[0]$. This issue could be resolved by the *"zoom-out"* method in [12], whose main idea is that, at the beginning of quantization, the encoder and the decoder find a large enough set which the state fall in.

Remark: For a scalar MJLS ($n = 1$), it can be seen that the necessary condition in eq. 13 (eq. 12) is actually the same as the sufficient condition in eq. 15. So the minimum constant bit rate $R = R_0$ has been achieved, which agrees with [18]. For multi-dimensional systems, the sufficient condition in eq. 15 usually requires a higher bit rate R than the necessary condition in 13 (eq. 12). How to narrow the gap between these two conditions is one of future research directions.

B. An example

Consider an example MJLS with $N = 2$, $n = 2$, $A_1 = \begin{bmatrix} 0 & 1 \\ 1.8 & -0.3 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0.7 & 1 \\ 1.8 & -0.3 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $K_1 = \begin{bmatrix} -0.8846 & -0.3611 \end{bmatrix}$, $K_2 = \begin{bmatrix} -1.8 & 1.1507 \end{bmatrix}$ and $T = \begin{bmatrix} 0.1 & 0.9 \\ 0.3 & 0.7 \end{bmatrix}$. We get $\lambda_{max}(\underline{S}) = 1.96$. According to Theorem 3.2, we know any stabilizing R must satisfy

$$R \geq 1. \quad (26)$$

We know $\lambda_{max}(\overline{S}) = 3.10$. So $R_0 = 1$ by eq. 16. According to Theorem 4.1, we know the quantized MJLS can be stabilized if

$$R \geq nR_0 = 2. \quad (27)$$

So we choose $R = 2$ (bits/step). 10000 sample paths are run to compute $\mathbf{E}\left[\|x[k]\|_2^2\right]$, $\mathbf{E}\left[\|e[k]\|_2^2\right]$ and $\mathbf{E}\left[\|L[k]\|_2^2\right]$, which are shown in Fig. 2.

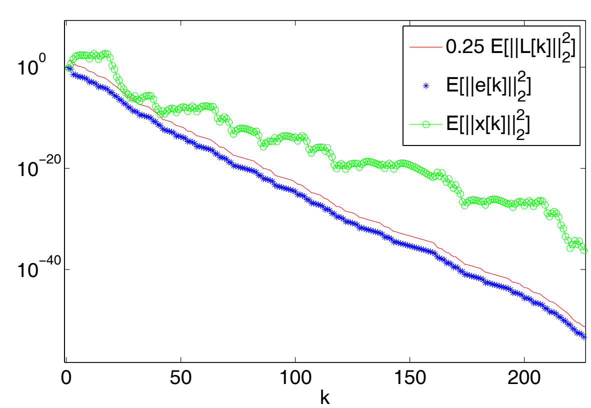

Fig. 2. Simulations results

From Fig. 2, we observe that

1) $\mathbf{E}\left[\|e[k]\|_2^2\right]$ is not larger than $0.25\mathbf{E}\left[\|L[k]\|_2^2\right]$, which confirms the no-overflow assertion of Proposition 4.3;
2) $\mathbf{E}\left[\|L[k]\|_2^2\right]$ converges to 0 as predicted by Proposition 4.4;

3) $\mathbf{E}\left[\|x[k]\|_2^2\right]$ converges to 0, which verifies the mean square stability of the quantized MJLS in eq. 2 of Theorem 4.1.

By Corollary 4.2, we know the minimum stabilizing bit rate R_{min} satisfies

$$1 \leq R_{min} \leq 2.$$

In this example, we get a very narrow range of R_{min}.

V. Conclusions

In this paper, we investigate the relationship between the mean square stability and the constant feedback bit rate of a quantized Markov jump linear system. We propose a lower bit rate bound, below which stability cannot be achieved. We also propose a quantization policy and show that policy can stabilize the system when the feedback bit rate is above some level. Unfortunately there is a gap between the achieved necessary and sufficient bit rate conditions (to guarantee the system's stability). How to narrow that gap will be one of future research directions.

References

[1] M. Mariton, *Jump Linear Systems in Automatic Control*. Marcel Dekker, Inc., 1990.

[2] Y. Ji, H. Chizeck, X. Feng, and K. Loparo, "Stability and control of discrete-time jump linear systems," *Control Theory and Advanced Technology*, vol. 7(2), pp. 247–270, 1991.

[3] O. Costa, M. Fragoso, and R. Marques, *Discrete-time Markov jump linear systems*. Springer, 2005.

[4] O. Beldiman, G. Walsh, and L. Bushnell, "Predictors for networked control systems," in *American Control Conference*, 2000, pp. 2347–2351.

[5] G. N. Nair, F. Fagnani, S. Zampieri, and R. J. Evans, "Feedback control under data rate constraints: an overview," *IEEE Proceedings*, vol. 95(1), pp. 108–137, 2007.

[6] N. Elia and S. Mitter, "Stabilization of linear systems with limited information," *IEEE Transactions on Automatic Control*, vol. 46(9), pp. 1384 –1400, 2001.

[7] M. Fu and L. Xie, "The sector bound approach to quantized feedback control," *IEEE Transactions on Automatic Control*, vol. 50(11), pp. 1698–1711, 2005.

[8] D. Delchamps, "Stabilizing a linear system with quantized state feedback," *IEEE Transactions on Automatic Control*, vol. 35(8), pp. 916–924, 1990.

[9] J. Baillieul, "Feedback designs in information-based control," *Lecture Notes in Control and Information Sciences*, vol. 280, pp. 35–37, 2002.

[10] F. Fagnani and S. Zampieri, "Stability analysis and synthesis for scalar linear systems with a quantized feedback," *IEEE Transactions on Automatic Control*, vol. 48(9), pp. 1569–1584, 2003.

[11] H. Ishii and B. Francis, "Quadratic stabilization of sampled-data systems with quantization," *Automatica*, vol. 39(10), pp. 1793–1800, 2003.

[12] R. Brockett and D. Liberzon, "Quantized feedback stabilization of linear systems," *IEEE Transactions on Automatic Control*, vol. 45(7), pp. 1279 –1289, 2000.

[13] D. Liberzon, "On stabilization of linear systems with limited information," *IEEE Transactions on Automatic Control*, vol. 48(2), pp. 304–307, 2003.

[14] S. Tatikonda and S. Mitter, "Control under communication constraints," *IEEE Transactions on Automatic Control*, vol. 49(7), pp. 1056–1068, 2004.

[15] G. N. Nair and R. Evans, "Exponential stabilisability of finite-dimensional linear systems with limited data rates," *Automatica*, vol. 39, pp. 585–593, 2003.

[16] C. Zhang, K. Chen, and G. E. Dullerud, "Stabilization of markovian jump linear systems with limited information - a convex approach," in *American Control Conference*, 2009, pp. 4013–4019.

[17] N. Xiao, L. Xie, and M. Fu, "Quantized stabilization of markov jump linear systems via state feedback," in *American Control Conference*, 2009, pp. 4020–4025.

[18] G. N. Nair, S. Dey, and R. Evans, "Communication-limited stabilisability of jump markov linear systems," in *15th Int. Symp. Math. Theo. Netw. Sys.*, 2002.

[19] G. N. Nair, S. Dey, and R. J. Evans, "Infimum data rates for stabilising markov jump linear systems," in *IEEE Control and Decision Conference*, 2003, pp. 1176–1181.

[20] O. Costa and M. Fragoso, "Stability results for discrete-time linear systems with markovian jumping parameters," *Journal of Mathematical Analysis and Applications*, vol. 179, pp. 154–178, 1993.

[21] R. Horn and C. Johnson, *Matrix analysis*. Cambridge University Press, 1985.