Constrained Optimal Control of Linear and Hybrid Systems—
Francesco Borelli (Berlin, Germany: Springer-Verlag, 2003). Reviewed
by Hai Lin

I. INTRODUCTION

Nearly every industrial application of automatic control imposes
constraints, which may be caused by, for example, actuator saturations,
restricted operational regions, or safety considerations. In addition,
some control performance requirements can be transformed into cer-
tain constraints in the state–space. Unfortunately, systematic methods
for handling constrained control problems are relatively sparse,
and many industrial applications often resort to ad-hoc approaches.
Therefore, the last decades have seen increasing research activities
in analysis and design of constrained control systems; see, for example,
[3] and [5]–[7].

One of the most popular approaches for dealing with control prob-
lems with constraints is model predictive control (MPC). MPC falls
into the category of sampled data control. However, instead of pre-
computing a feedback control law or control sequence for the plant
as in conventional control, MPC determines the current control action
by solving, at each sampling instant, a finite time open-loop optimal
control problem, using the current state of the plant as the initial state.
The input, output, and state constraints, together with additive distur-
bances and parametric uncertainties, can be easily handled within the
MPC framework by incorporating these constraints into the optimiza-
tion problem at each sampling instant. This advantage makes MPC very
popular in the process industry where satisfaction of constraints is par-
ticularly important [7].

Since MPC needs to solve an online optimization problem during
each sampling period, it is necessary that the plants being controlled
are sufficiently “slow” to permit its implementation. This is the biggest
drawback of MPC and limits its applicability to relatively slow and/or
small problems. This motivates research efforts with the aim to move
all the computations necessary for the implementation of MPC offline.
These efforts should largely increase the range of applicability of MPC
to industrial problems. The book by Borelli reports the most recent
progress achieved by the author and his collaborators along this direc-
tion via using multiparametric programming techniques.

Multiparametric programming is a kind of mathematical program
dependent on a vector of parameters, which may appear in the cost
function and/or in the constraints. The solution of a multiparametric
programming is characterized as a function that maps feasible param-
ters onto optimization solvers. The multiparametric programming is of
interest here, since the discrete-time finite time open-loop constrained
optimal control problem can be formulated as a multiparametric pro-
gram, where the sequence of control inputs are the optimization vari-
able and the initial states are the parameters. The advantage of this
method is that the multiparametric program can be solved offline and
induce a feedback control law for the full range of initial states (instead
of a sequence of control signals for a specific initial condition). This
provides attractive features for MPC on two accounts. First, it will al-
leviate the online computation burden, since only function evaluations
are necessary at each sampling instant. Second, it also provides an in-
sight into the structure underlying optimization-based controllers. It is
interesting to notice that the optimal control law is usually character-
ized as a piecewise affine state feedback with polyhedral partition of
the state–space.

II. THE BOOK

The book focuses on two classes of discrete-time dynamical systems,
namely constrained linear systems and linear hybrid systems. Hybrid
systems are heterogeneous dynamical systems, the behavior of which
is determined by interacting continuous-variable and discrete-event dy-
namics [2]. Linear hybrid systems are referred to as classes of hybrid
systems whose continuous-variable dynamics are described by linear
differential/difference equations. Hybrid systems have been identified
in a wide variety of applications in control of mechanical systems,
process control, automotive industry, power systems, aircraft and traffic
control, among many other fields. Interested readers may refer to spe-

Several kinds of constrained optimal control problems with cost
functions based on 1, 2, or ∞ norms for linear systems and linear
hybrid systems are investigated in the book. The basic idea is to for-
mulate the constrained optimal control problems into corresponding
multiparametric programs, and to characterize the explicit optimal
feedback control laws via multiparametric programming. The book is
organized into four parts.

The first part of the book contains some preliminary results on
multiparametric programming. The author begins with an introd-
cution of basic theory for general nonlinear parametric programming.
Then, three efficient geometric algorithms are developed for solving
multiparametric linear programs (mp-LP), multiparametric quadratic
programs (mp-QP), and multiparametric mixed-integer linear programs
(mp-MILP), respectively. The basic idea of the algorithms is to con-
struct a neighborhood region of a given parameter in the parameter
space, by using necessary and sufficient conditions for optimality, and
then recursively explore the parameter space outside such a region. It
is interesting that this kind of region, called critical region, is usually
characterized as a polyhedron.

In the second part, the author studies three kinds of constrained op-
timal control problems for discrete-time linear systems with polyhedral
constraints on inputs and states. First, in Chapter 2, the constrained fi-
nite time optimal control problems with cost functions based on 1, 2,
or ∞ norms are formulated as multiparametric linear (1 or ∞ norm
based) or multiparametric quadratic (2-norm based) programs. The fi-
nite length input sequence is the optimization vector, while the cur-
rent state of the dynamical systems enters the cost function and the
constraints as the parameter vector. It is demonstrated that the solution
to all these optimal control problems can be expressed as a piecewise
affine state feedback law, accompanied with a polyhedral partition of
the state space. Moreover, the optimal control law is continuous, and
the value function is convex and continuous. Second, the infinite time
constrained optimal control problems with 2-norm based cost func-
tion, namely constrained linear quadratic regulator (CLQR) problem,
is considered in Chapter 3. The solution for CLQR is given or approx-
imated by a finite time constrained optimal control with sufficiently
long horizon.

Since the multiparametric programs can be solved offline, the im-
plementation of MPC is reduced to evaluating the precomputed explicit
piecewise affine feedback control law based on current state measurement. Therefore, the characterization of explicit feedback controls for finite time optimal control problems have important consequences for the implementation of MPC. Chapter 4 focuses on MPC problem for constrained linear systems and briefly discusses its stability.

The robust optimal control problem is also considered for constrained linear systems with additive norm-bounded input disturbances and polytopic parametric uncertainties in Chapter 5. It is shown that the robust optimal control law over a finite horizon is a continuous piecewise affine function of the state and the value function is convex and continuous. The piecewise affine feedback control law is based on the polyhedral partition of the state–space, and the number of the polyhedral regions could increase dramatically with the number of constraints in the optimal control problem. Therefore, the author develops two algorithms in Chapter 6 to reduce the storage demands and computational complexity.

The third part (Chapters 7 and 8) of the book is on constrained optimal control for linear hybrid systems. After a short overview of different models for linear hybrid systems, the author uses a piecewise affine (PWA) modeling framework to study the state feedback solution to the finite time optimal control problem for hybrid systems, which can be computed by means of multiparametric mixed integer programming. The optimal control law is shown to be, in general, piecewise affine over nonconvex and disconnected sets. However, this method is basically based on the enumeration of all possible switching sequences, the number of which grows exponentially with the time horizon. Therefore, another hybrid modeling framework, mixed logic dynamical (MLD) system, is employed to avoid such enumeration problems. The key idea of MLD systems is to express the logic relationship in the hybrid systems’ description into mixed integer inequalities by transforming Boolean variables into 0–1 integers. It is worth pointing out that the MLD systems are equivalent to other classes of discrete-time hybrid systems such as PWA systems, linear complementarity (LC) systems, and max–min-plus-scaling (MMPS) systems [4]. The MPC problem is also studied for both PWA and MLD systems.

Finally, the applicability of the theoretical results is demonstrated on a mechanical laboratory process and a traction control systems in the application part.

III. Comments

The book is a revised version of the author’s Ph.D. dissertation under the guidance of Dr. M. Morari and Dr. A. Bemporad from the Automatic Control Laboratory at ETH-Zurich, Zurich, Switzerland. All the main results presented in the book are contributed originally by the author and his collaborators. The style of the book is clear and concise with many illustrative examples.

Results in the book make interesting contributions to the research areas of constrained optimal control, MPC, and hybrid optimal control. Therefore, the book provides a valuable resource for researchers from both academia and industry. The book is also of interest to the operations research community, since efficient geometric algorithms are developed for multiparametric linear, quadratic, and mixed integer programming. However, some chapters in this book are very short, indicating potential for further development. In this regard, it would be of great interest to include sections discussing future research topics.

Finally, the main goal of the book is the development of efficient geometric algorithms for multiparametric programming and synthesis of explicit feedback control for constrained optimal control problems. Therefore, it would be of great value to develop a software toolbox for calculating the optimization solver.

REFERENCES