Switched Linear Systems: Control and Design—Z. Sun and S. S. Ge (London: Springer-Verlag, 2005). *Reviewed by Hai Lin*

I. Introduction

Traditionally, most of the research work in systems and control has been concerned predominantly with dynamical systems that are described purely as either time-driven continuous variable dynamics or event-driven discrete logic dynamics. Yet, there are numerous dynamical systems that are of heterogeneous nature. By heterogeneity, we mean that both continuous variable and discrete-event dynamics are present and interacting with each other to generate complex dynamical behaviors. These are termed hybrid systems and have been identified in a wide variety of natural and man-made systems, for example gene regulatory networks, embedded systems, process control, communication networks, aircraft and traffic control, and among many other fields.

To study such kind of heterogeneous dynamical systems, one needs to develop new methods, since the traditional approaches may become unsuitable. For example, if one intends to capture, or approximate, the hybrid phenomena, like jumping and switching, via a continuous variable model, then it may result in a very complicated differential equation that is difficult to handle. In contrast, the discrete models might be too abstract for some applications, especially when the plant to be controlled has tight interactions of continuous variable and discrete event dynamics, or when the requirements are represented by time and event-based behaviors.

The history of hybrid system research can be traced back at least to the 1950s with the study of engineering systems that contain relays and/or hysteresis. However, hybrid systems began to attract people's attentions in the early 1990s, mainly because of the vast development and implementation of digital micro controllers and embedded devices. The last decade has seen considerable research activities in modeling, analysis and synthesis of hybrid systems involving researchers from a number of traditionally distinct fields, such as computer science and control system engineering [3], [2]. Therefore, it is not surprising that there are a variety of approaches for modeling, analysis, and synthesis of hybrid systems.

On the one hand, computer scientists look at hybrid systems primarily as discrete (computer) programs interacting with the analog environment. They extend their computational models, such as finite state machine, automaton and Petri net, from discrete systems to hybrid systems by embedding the continuous variable dynamics into these discrete models. Typically, these approaches are able to deal with complex discrete dynamics and emphasize analysis results (verification) and simulation methodologies. On the other hand, researchers from the areas of dynamical systems and control theory extend the models and methodologies for traditional continuous variable systems, such as ordinary differential/difference equations, by including discrete variables so as to describe the jumping or switching phenomena. Typically, these approaches are able to deal with complex continuous variable dynamics and mainly concern stability, robustness and synthesis issues. This book falls into the second category and focuses on control and design for switched linear systems.

II. The Book

Switched linear system is a special class of hybrid systems, which contains a finite collection of linear time-invariant (LTI) subsystems and a switching law orchestrating switching between these subsystems. The book presents rigorous results on several fundamental control problems for switched linear systems, which include controllability, observability, stabilization and optimal control.

The first two chapters provide introductory mathematical preliminaries, background and motivating examples. It is interesting that the authors give a detailed classification of switching signals and operations on them, which provide a unified terminology throughout the book. This could be introduced into the switched system community and might foster a standardization of notations.

The first problem being addressed in the book is the stabilization problem. The stability study of switched systems can be roughly divided into two kinds of problems. One is the stability analysis of switched systems under given switching signals (maybe arbitrary, slow switching, etc.); the other is the synthesis of stabilizing switching signals for a given collection of dynamical systems. The stability analysis problem has been extensively studied during the last decade, and powerful tools like common Lyapunov function and multiple Lyapunov functions have been developed in [4]–[6]. On the contrary, relatively less attentions have been paid to the stabilization problem for switched systems until recently. In Chapter 3, the authors investigate the switching stabilization problems for switched systems with autonomous LTI subsystems. Under the assumption that there exists a stable convex combination of the subsystems' state matrices, periodic switching laws are proposed to stabilize the switched systems based on average techniques. Also, a state-feedback switching law is presented based on an appropriate partition of the state space, which represents extension of the "min-projection" approach in the literature. In addition, the switching frequency and robustness issues are considered. The most interesting part of this chapter is the proof for the equivalence of the various stabilizability concepts for switched linear systems. This represents a significant step towards understanding the switching stabilizability problem, which is an elusive open problem in the switched systems literature.

Chapter 4 is the highlight of the book, in which the controllability, observability, feedback equivalence and canonical forms for switched control systems are investigated. Controllability and observability for linear systems are bread-and-butter for our control community. However, no general results exist for nonlinear systems yet. The authors elegantly extended these concepts to switched linear systems, and developed nice geometric characterizations for the controllable/unobservable subspaces. For continuous-time switched linear systems, it turns out that the controllable subspaces share very similar geometric characterizations as linear systems, which are the minimum A-invariant subspace while containing the image space of B. Based on this, verifiable geometric criteria are presented for the controllability and observability. An important implication from this result is that the reachability/controllability problem for continuous-time switched linear systems is decidable. It is known that the reachability problem is undecidable for general hybrid systems [1], and the undecidability has been the main deterrent for the implementation of hybrid system theory to solve real world problems. This decidability nature for switched system makes it a very interesting and special subclass of hybrid systems. Switched system could model a large class of problems of practical importance, while it has relatively simple discrete-event dynamics so one may obtain decidable results for certain problems.

Another very interesting result in Chapter 4 is the decomposition of a switched linear system into a controllable part and an uncontrollable part as well as an observable part and an unobservable part. This gives us better understanding of the structural properties of the switched linear systems, and also paves the way for solving feedback stabilization problem in Chapter 5. The feedback stabilization considered in Chapter 5 is different from the switching stabilization problem

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in Chapter 3 in the following sense. In feedback stabilization, one needs to design both state/output feedback continuous control law and the switching signals, while, in Chapter 3, only switching signals are design variables. Therefore, the feedback stabilization problem is more challenging. The authors developed some preliminary but insightful results for certain special cases based on the canonical decomposition.

Chapter 6 addresses optimal control for switched linear systems, which is an active research field with many challenging problems. The basic challenge possibly comes from the non-convex nature of the optimization problem for switched systems. The authors considered two optimization problems for switched linear systems. The first is the optimal switching control to minimize the convergence rate. Second, a two-stage optimization methodology is developed for solving the mixed optimal switching/control problem of the switched linear system where both the switching signal and the control input are design variables.

Finally, Chapter 7 concludes the book by briefly summarizing the main results in the book and describing some research problems for further investigation.

III. Comments

The style of the book is clear and concise with many illustrative examples. The main results presented in the book are original contributions of the authors, and represent the culmination of years of rigorous and insightful research. Therefore, the book provides a valuable resource for researchers and engineers interested in switched systems. The book can also serve as a complementary reading for linear/nonlinear system theory at the graduate level.

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