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A Switched System Approach to Scheduling of Networked Control Systems with Communication Constraints

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Abstract— **This paper presents a scheduling strategy for a collection of discrete-time networked control systems (NCSs) subjected to communication constraints. Communication constraints under consideration include medium access constraint, network-induced delays and packet-dropouts. A feedback control system with the communication constraints is modelled as a switched delay system which switches on the open-loop and closed-loop models according to whether the feedback control loop gains access to the network or not. Delay-dependent sufficient conditions for exponential stability with L_2 gain performance are developed for the switched system. Based on the stability conditions, sufficient conditions are presented on the existence of scheduling policy that simultaneously stabilizes the collection of NCSs. Simulation on network-based control of unstable batch reactor systems is performed to demonstrated the effectiveness of the proposed scheduling strategy.**

I. Introduction

Shared communication network is increasingly being used to support information exchange in control of a group of spatially distributed systems. This is very typical setups when using the base station to control and coordinate multiple mobile robots [1], unmanned aerial vehicles (UAVs) and autonomous air vehicles (AAVs) [2], through a wireless/wired network. Control systems whose control loops are closed over a wired or wireless communication network are known as networked control systems (NCSs). Motivation for using communication network in control comes from higher system testability and resource utilization, as well as lower cost, reduced weight and power, simpler installation and maintenance [3]. However, many practical networks are subjected to communication constraints that make the analysis and design of NCSs be typically harder than that of classical control systems. NCSs require novel control design methodologies that differ in nature from the tradition control viewpoint to account for the presence of communication constraints in the closed-loops. For example, (i) data packets transmitted over networks are usually subjected to random or time-varying delays, and/or even may be lost during the information transmission, see for example the recent servery paper [4] and references listed therein; and (ii) because of the limitation on channel capacity, the shared communication medium imposes access constraints: only a limited number of the network nodes are allowed to transmit their data packets at any time instant [3], [5], [6], [7].

Simultaneous stability analysis for a group of NCSs has recently received increasing attention [5], [6], [8], [9], [10]. In the NCSs setting, medium access constraint was introduced by Hristu-Varsakelis in [5], [6], where only a few of the family of plants can gain the shared network access to exchange information with their remotely located controllers at any one time, while others must wait. The medium access constraint raises the issue of communication scheduling strategies that determine how and when each control loop gains access to the network. A static periodic network allocation sequence was presented in [5] for simultaneous stability of the collection of linear systems, and the network allocation policy was improved in [8] with the introduction of an interrupt-based communication strategy. In [9], the rate monotonic scheduling algorithm was applied to schedule a set of NCSs. A time-division scheduling policy was developed in [10] by employing average dwell time technique combined with piecewise Lyapunov-like functions. These studies are carried out under the assumption that data packets are transmitted over an idealized shared network such that network-induced delays and data packet dropouts are not present in the network. In [11], network-induced delays less than a sample period were modelled as polytopic uncertainties in a sample-data control framework and robust control methods were applied to the polytopic systems.

In this paper, medium access constraint introduced in [5], [6], network-induced delays, and data packet dropouts are simultaneously taken into account in the shared communication network. The collection of NCSs under consideration is modelled as a group of switched systems with time-varying delays. The modelling approach makes us apply the ideas already developed in the switched system framework [12], [13], [14] for the simultaneous stability analysis for the NCSs in the presence of communication constraints. Hoverer, the existing results on the stability of switched delay systems cannot be directly applied to analyse the stability of NCSs. The main contributions of the paper include: (i) delay-dependent sufficient conditions for exponential stability with weighted L_2 gain are presented for the switched delayed systems composed with both stable and unstable subsystems; and (ii) simultaneous stability is developed for the NCSs under a static periodic scheduling policy.

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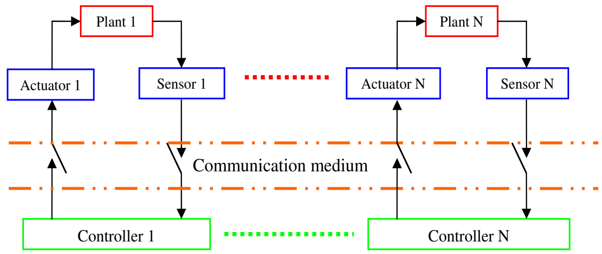

Fig. 1. Communication of controller-plants through a shared communication medium.

II. NCS Model

In this paper, we consider the NCSs consisting of a collection of discrete-time linear plants whose feedback control loops are closed via a shared communication network, as illustrated in Fig. 1. The dynamics of the i-th plant, $i \in \mathcal{N} = \{1, ..., N\}$, is given by

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + G_i \omega_i(k) \quad (1)$$
$$z_i(k) = C_i x_i(k) + D_i \omega_i(k) \quad (2)$$

where $x_i(k) \in \mathbb{R}^{n_i}$ are the system states, $u_i(t) \in \mathbb{R}^{r_i}$ are the control inputs, $z_i(k) \in \mathbb{R}^{l_i}$ are the controlled outputs, and $\omega_i(k) \in \mathbb{R}^{p_i}$ are the disturbance inputs, which belong to $L_2[0, \infty)$.

Consider state-feedback network-based controllers:

$$u_i(k) = K_i x_i(k). \quad (3)$$

Assumption 1: [5], [6] Because of limited communication capacities, not all the control loops in the NCSs can be addressed at the same time. At each discrete-time instant, only C_{max} of the N plants ($C_{max} < N$) are allowed to communicate with their remote controllers while others must wait.

Assumption 2: [5], [6] When a plant fails to communicate with its corresponding controller, the open-loop system might be unstable; otherwise, the plant gains access to communicate with its controller and the resulting closed-loop system is stable.

Assuming that the network communication is unreliable, there may exist transmission delays and packet-dropouts in practical communication channel. Let $\mathcal{P} = \{q_1, q_2, \ldots\}$ be a subsequence of $1, 2, 3 \ldots$, which denotes the sequence of time points of successful data transmissions from the sensors to actuators. Let $\bar{\xi}_i = \max_{q_k \in \mathcal{P}}\{\xi_i(q_k)\}$ be the maximum packet-dropout upper bounds, where $\xi_i(q_k)$ are the number of accumulated data packet dropouts from the last updating instant q_k to the next updating instant q_{k+1}.

Transmission delay: It is supposed that the signal transmission delays $\tau_i(q_k)$ satisfy

$$\underline{\tau}_i \leq \tau_i(q_k) \leq \overline{\tau}_i \quad (4)$$

where $\underline{\tau}_i$ and $\overline{\tau}_i$ are constant positive scalars representing the lower and upper bounds of transmission delays in the network, respectively. From the viewpoint of the zero-hold, the control input is given by

$$u_i(k) = K_i x_i(q_k - \tau_i(q_k)), \; q_k \leq k \leq q_{k+1} - 1 \quad (5)$$

where q_{k+1} is the next updating instant of the actuators after q_k, and the initial condition of the control input is set to zero, i.e., $u_i(l) = 0, \; 0 \leq l \leq q_1 - 1$.

Packet-dropout: Note that $\bar{\xi}_i = \max_{q_k \in \mathcal{P}}\{\xi_i(q_k)\}$ are the maximum packet-dropout upper bounds, which gives

$$0 \leq \xi_i(q_k) \leq \bar{\xi}_i \quad (6)$$

According to the above analysis, and inequalities (4) and (6), it can be seen that

$$1 \leq q_{k+1} - q_k \leq \bar{\xi}_i + 1 + \overline{\tau}_i \quad (7)$$

which implies that the interval between any two successive updating instants is upper bounded by $\bar{\xi}_i + 1 + \overline{\tau}_i$. Based on the above analysis, systems (1) with (5) can be described by

$$x_i(k+1) = A_i x_i(k) + B_i K_i x_i(q_k - \tau_i(k)) + G_i \omega_i(k) \quad (8)$$

where $q_k \leq k \leq q_{k+1} - 1$, $q_k \in \mathcal{P}$. Let $d_i(k) = k - q_k + \tau_i(q_k)$, thus it follows that $q_k - \tau_i(q_k)$ in (8) can expressed as

$$q_k - \tau_i(q_k) = k - d_i(k) \quad (9)$$

From (7), it can be seen that

$$\underline{d}_i \leq d_i(k) \leq \overline{d}_i \quad (10)$$

where $\underline{d}_i = \underline{\tau}_i$ and $\overline{d}_i = 2\overline{\tau}_i + \bar{\xi}_i$. Substituting (9) into (8), it follows

$$x_i(k+1) = A_i x_i(k) + B_i K_i x_i(k - d_i(k)) + G_i \omega_i(k) \quad (11)$$

Under Assumption 1, some control loops of the plants are open for some time because the shared network link is occupied by another users. Thus, the i-th control system (11), $i \in \mathcal{N} = \{1, ..., N\}$ switches on its open-loop and closed-loop status, which can be described by the following switched system:

$$x_i(k+1) = A_{i\sigma_i(k)} x_i(k) + A_{di\sigma_i(k)} x_i(k - d_i(k)) + G_{i\sigma_i(k)} \omega_i(k) \quad (12)$$

where the switching signals $\sigma_i(k) : \mathbb{Z}_+ = \{0, 1, 2, \ldots\} \to \{1, 2\}$ are piecewise constant functions of time k. Let subsystem "1" denote the case of the closed-loop status, and subsystem "2" describe the control loop being open, i.e.,

$$A_{i1} = A_i, \; A_{di1} = B_i K_i, \; G_{i1} = G_i \quad (13)$$
$$A_{i2} = A_i, \; A_{di2} = 0, \; G_{i2} = G_i \quad (14)$$

Remark 1: In practical applications, it might be desirable to make control components like sensors, actuators, and controllers work intermittently. For example, control components are suspended from time to time for an economic or system life under consideration [15]. On the other hand, occasional failures of feedback control often occur in real-world applications since control signals are not transmitted perfectly or control components are complete outage [15]. In these cases, the control system with intermittent feedback could be described by a switched system and the switching signal depends on whether the controller works or not.

Under Assumptions 1-2 and considering the transmission delay and packet-dropout effects, the objective of the paper is to design a scheduling policy such that all systems in (1) and (2) are exponentially stabilized with L_2 gain performance.

III. Stability of Switched Systems

This section dedicates to deriving delay-dependent stability conditions for discrete-time switched systems with time-varying delays. In Section II, a single control system is modelled as a switched delay system composed of stable closed-loop subsystem and unstable open-loop subsystem. Recently, switched Lyapunov function method was extensively used to assess asymptotic stability of discrete-time switched delay systems [16], [17], [18], where arbitrary switching signal was considered and thus every subsystem in switched systems was required to be stable. So far, few result has appeared to test asymptotic stability for discrete-time switched delay systems with unstable subsystems [19]. In [19], average dwell time technique [20] was utilized to design a stabilizing switching signal for delay switched systems with unstable subsystem, where the delay was assumed to be constant. But, the delays involved in system (12) are time-varying, thus, these existing results cannot be used to analyse the stability of system (12). For the sake of brevity, the subscript i will be dropped in the section.

Definition 1: [10] For any discrete-time instant $k > 0$, let $\alpha_c(k)$ denote the total time interval of the plant being closed-loop (attended by the controller) during $[0\ k)$, and the ratio $\frac{\alpha_c(k)}{k}$ is said to be the attention rate of the plant, and let $N(k)$ denote the total number of switchings between closed-loop with open-loop during $[0\ k)$, which is said to be the attention frequency.

Consider the subsystem of the switched delay system (12) given by

$$x(k+1) = A_j x(k) + A_{dj} x(k - d(k)) + G_j \omega(k). \quad (15)$$

For system (15), choose the following positive definite quadratic functionals:

$$\begin{aligned} V_j(k) &= V_{1j}(k) + V_{2j}(k) + V_{3j}(k) + V_{4j}(k) \\ V_{1j}(k) &= x^T(k) P_j x(k) \\ V_{2j}(k) &= \sum_{\delta=-\bar{d}+1}^{0} \sum_{l=k-1+\delta}^{k-1} y^T(l) \lambda_j^{k-1-l} Z_{1j} y(l) \\ &\quad + \sum_{\delta=-\bar{d}+1}^{-\underline{d}} \sum_{l=k-1+\delta}^{k-1} y^T(l) \lambda_j^{k-1-l} Z_{2j} y(l) \\ V_{3j}(k) &= \sum_{l=k-\underline{d}}^{k-1} x^T(l) \lambda_j^{k-1-l} Q_{1j} x(l) \\ &\quad + \sum_{l=k-\bar{d}}^{k-1} x^T(l) \lambda_j^{k-1-l} Q_{2j} x(l) \quad (16) \\ V_{4j}(k) &= \sum_{\delta=-\bar{d}+1}^{-\underline{d}+1} \sum_{l=k+\delta}^{k-1} x^T(l) \lambda_j^{k-1-l} Q_{3j} x(l) \\ y(l) &= x(l+1) - x(l) \end{aligned}$$

where $\lambda_j > 0$, $P_j > 0$, $Q_{1j} \geq 0$, $Q_{2j} \geq 0$, $Q_{3j} \geq 0$, $Z_{1j} > 0$, $Z_{2j} > 0$, $j \in \{1, 2\}$.

It should be mentioned that the positive definite quadratic functional (16) is similar to the Lyapunov-Krasovskii functional in [21], i.e., if $\lambda_j = 1$, functional (16) shrinks to the one in [21] where it is used to derive asymptotic stability conditions for non-switched delay system. Along any state trajectory of system (15), an exponential decay or increase estimate of $V_j(k)$ in (16) is firstly presented in the following lemma, which plays an important role in the development.

Lemma 1: For given scalars $\gamma_j > 0$, $\lambda_j > 0$ and $\bar{d} \geq \underline{d} \geq 0$, if there exist matrices $P_j > 0$, $Q_{1j} \geq 0$, $Q_{2j} \geq 0$, $Q_{3j} \geq 0$, $Z_{1j} > 0$, $Z_{2j} > 0$, $V_j = [V_{1j}^T\ V_{2j}^T\ V_{3j}^T\ V_{4j}^T\ V_{5j}^T]^T$, $W_j = [W_{1j}^T\ W_{2j}^T\ W_{3j}^T\ W_{4j}^T\ W_{5j}^T]^T$, $S_j = [S_{1j}^T\ S_{2j}^T\ S_{3j}^T\ S_{4j}^T\ S_{5j}^T]^T$, $j \in \{1, 2\}$, such that the following inequalities

$$\begin{bmatrix} \Phi_j & \Pi_j \\ * & \Omega_j \end{bmatrix} < 0, \quad j \in \{1, 2\} \quad (17)$$

hold, then along any state trajectory of system (15) with the time-varying delay (9), the following inequalities satisfy

$$V_j(k+1) \leq \lambda_j V_j(k) - \Gamma_j(k) \quad (18)$$

where

$$\Gamma_j(k) = z^T(k) z(k) - \gamma_j^2 \omega^T(k) \omega(k), \quad (19)$$

$*$ denotes the symmetric terms in a symmetric matrix, and

$$\Phi_j = \begin{bmatrix} \Phi_{11j} & \Phi_{12j} & \Phi_{13j} & \Phi_{14j} & \Phi_{15j} \\ * & \Phi_{22j} & \Phi_{23j} & \Phi_{24j} & \Phi_{25j} \\ * & * & \Phi_{33j} & \Phi_{34j} & S_{5j}^T \\ * & * & * & \Phi_{44j} & -W_{5j}^T \\ * & * & * & * & \Phi_{55j} \end{bmatrix},$$

$$\begin{aligned} \Pi_j &= [\Xi_{1j}^T P_j\ \ \bar{d}\Xi_{2j}^T Z_{1j}\ \ \tilde{d}\Xi_{2j}^T Z_{2j}\ \ c_{1j} V_j\ \ c_{2j} W_j\ \ c_{3j} S_j], \\ \Omega_j &= \mathrm{diag}\{-P_j\ \ -\bar{d} Z_{1j}\ \ -\tilde{d} Z_{2j}\ \ -c_{1j} Z_{1j} \\ &\qquad -c_{2j} Z_{2j}\ \ -c_{3j} Z_{3j}\}, \end{aligned}$$

$\Xi_{1j} = [A_j\ A_{dj}\ 0\ 0\ G_j]$, $\Xi_{2j} = [A_j - I\ A_{dj}\ 0\ 0\ G_j]$,
$\Phi_{11j} = -\lambda_j P_j + Q_{1j} + Q_{2j} + (\tilde{d}+1) Q_{3j} + V_{1j} + V_{1j}^T + C^T C$,
$\Phi_{12j} = -V_{1j} + V_{2j}^T + W_{1j} - S_{1j}$, $\Phi_{13j} = V_{3j}^T + S_{1j}$,
$\Phi_{14j} = V_{4j}^T - W_{1j}$, $\Phi_{15j} = V_{5j}^T + C^T D$,
$\Phi_{22j} = -\lambda_j^{\bar{d}} Q_{3j} - V_{2j} - V_{2j}^T + W_{2j} + W_{2j}^T - S_{2j} - S_{2j}^T$ (for $0 < \lambda_j \leq 1$), or
$\Phi_{22j} = -\lambda_j^{\underline{d}} Q_{3j} - V_{2j} - V_{2j}^T + W_{2j} + W_{2j}^T - S_{2j} - S_{2j}^T$ (for $\lambda_j \geq 1$),
$\Phi_{23j} = S_{2j} - V_{3j}^T + W_{3j}^T - S_{3j}^T$,
$\Phi_{24j} = -V_{4j}^T - W_{2j}^T - S_{4j}^T + W_{4j}^T$, $\Phi_{25j} = -V_{5j}^T + W_{5j}^T - S_{5j}^T$,
$\Phi_{33j} = -\lambda_j^{\underline{d}} Q_{1j} + S_{3jq} + S_{3jq}^T$, $\Phi_{34j} = -W_{3j} + S_{4j}^T$,
$\Phi_{44j} = -\lambda_j^{\bar{d}} Q_{2j} - W_{4j} - W_{4j}^T$, $\Phi_{55j} = -\gamma_j^2 I + D^T D$,
$Z_{3j} = Z_{1j} + Z_{2j}$, $\tilde{d} = \bar{d} - \underline{d}$, $c_{1j} = (\lambda_j^{-\bar{d}} - \lambda_j^2)/(1 - \lambda_j)$,
$c_{2j} = (\lambda_j^{-\bar{d}} - \lambda_j^{\underline{d}+1})/(1 - \lambda_j)$, $c_{3j} = \lambda_j^{\underline{d}+1} + \lambda_j^{\underline{d}}$.

Proof: The proof follows the similar approach as Theorem 1 in [21] and will be omitted for conciseness. ∎

Remark 2: By iterative substitutions, inequality (18) yields

$$V_j(k) \leq \lambda_j^{(k-k_0)} V(k_0) - \sum_{s=k_0}^{k-1} \lambda_j^{(k-s-1)} \Gamma_j(s) \quad (20)$$

where $\Gamma_j(s)$ is defined in (19), which implies that the functional $V_j(k)$ in (16) along any state trajectory of system (15) with $\omega(t) = 0$ has an exponential decay rate λ_j ($0 < \lambda_j < 1$) or increase rate λ_j ($\lambda_j > 1$). When $\lambda_j = 1$, inequality (17) gives delay-dependent sufficient conditions for asymptotical stability with L_2 gain γ_j for system (15).

Based on the above exponential decay or increase estimate of $V_j(k)$ in Lemma 1, the following result gives exponential stability and preserving weighted L_2 gain for system (12).

Lemma 2: Consider the switched delay system (12) and (2) with subsystem (13) and subsystem (14). Supposed that there exists a piecewise quadratic Lyapnov-like function candidate

$$V(k) = \begin{cases} V_1(k), & \text{if closed-loop} \\ V_2(k), & \text{if open-loop} \end{cases} \tag{21}$$

satisfying (18) and

$$V_1(k) \leq \mu V_2(k),\ V_2(k) \leq \mu d_\lambda V_1(k) \tag{22}$$

where $0 < \lambda_1 < 1$, $\lambda_2 > \lambda_1$, $\mu > 1$, $d_\lambda = (\frac{\lambda_2}{\lambda_1})^{\overline{d}-1} \geq 1$. Then the switched system (12) and (2) is exponentially stable with decay rate $0 < \rho < 1$ and weighted L_2 gain $\sqrt{c}\gamma_0$ ($\gamma_0 = \max_{j\in\{1,2\}} \gamma_j$) under the switching signal $\sigma(k)$ with the following conditions:

i) The attention rate satisfies

$$\frac{\alpha_c(k)}{k} \geq \frac{\ln\lambda_2 - \ln\lambda^*}{\ln\lambda_2 - \ln\lambda_1} \tag{23}$$

ii) The attention frequency satisfies

$$N(k) \leq N_0 + k/T_a,$$
$$N_0 = \frac{\ln c}{2\ln\mu + \ln d_\lambda},\ T_a > T_a^* = \frac{2\ln\mu + \ln d_\lambda}{2\ln\rho - \ln\lambda^*} \tag{24}$$

where T_a and N_0 are said to be the average dwell time and the chatter bound [20], respectively, $\lambda_1 < \lambda^* < \rho^2 < 1$, $c > 0$.

Proof: See Appendix A. ∎

IV. Scheduling

This section aims to provide a systematic method to find the policy for establishing and terminating communication between each system and its controller in a way that stabilizes all systems with preserving L_2 gains. To this end, the conception of "schedulability" is defined as follows.

Definition 2: Under Assumptions 1-2, a collection of NCSs sharing the limited network resource is said to be schedulable if there exists a scheduling policy such that all N systems are stabilized and have certain desired L_2 gains.

Lemma 3: Under Assumptions 1-2, consider the collection of NCSs (1) and (2) with communication constraints as shown in Fig. 1. the collection of NCSs is schedulable if the piecewise quadratic Lyapnov-like function (21) satisfies (18) and (22) for any individual control system, and the following condition holds:

$$\sum_{i=1}^{N} \frac{\ln\lambda_{i2}}{\ln\lambda_{i2} - \ln\lambda_{i1}} < C_{\max} \tag{25}$$

where $0 < \lambda_{i1} < 1$, $\lambda_{i2} > 1$, $C_{\max} < N$, and $C_{\max}$ denotes the maximum number of plants which can communicate with their remote controllers at any time instant. Moreover, the scheduling policy can be adopted in the following periodic scheduling way:

i) Choose $\mathcal{T} = \max_{1\leq i\leq N}\{L_i\}$, where L_i is a positive integer sufficiently large to satisfy the average dwell time condition in (24) for the i-th plant. For example, we may set $L_i = \lceil T_{ai}^* \rceil$, where T_{ai}^* is the lower bound of the average dwell time T_{ai}, and $\lceil\cdot\rceil$ denotes the upper integer bound.

ii) Close $C_{\max}$ control loops for their plants at any time instant. Activate the control loops from 1 to N in order, and let the i-th control loop work for a time interval of length $\lceil \beta_i \mathcal{T} \rceil$ for $i = 1, \ldots, N$, where the designed parameters can be chosen as

$$\beta_i = \frac{\ln\lambda_{i2} - \ln\lambda_i^*}{\ln\lambda_{i2} - \ln\lambda_{i1}},\ \lambda_i^* = \lambda_{i2}^{-\varepsilon},\ \lambda_{i2}^{-\varepsilon} < \rho^2 < 1,$$
$$\overline{\varepsilon} = 1 - \frac{1}{C_{\max}} \sum_{i=1}^{N} \frac{\ln\lambda_{i2}}{\ln\lambda_{i2} - \ln\lambda_{i1}},\ 0 < \varepsilon \leq \overline{\varepsilon} \tag{26}$$

Proof: The proof follows the constructive approaches as Theorem 2 in [10] and Lemma 2 in [11], and will be omitted for conciseness. ∎

Remark 3: The form of schedulable condition (25) is similar to the continuous-time one in [8], [6], [10], which only depends on the estimates for the convergence rate λ_{i1} of the i-th closed-loop system and the convergence/divergence rate λ_{i2} of the i-th open-loop plant. To obtain a large value of N, it is desirable that λ_{i1} and λ_{i2} are small. This is reasonable since small convergence rate ($\lambda_{i1} < 1$) means that the states of i-th closed-loop system converge fast when the plant be attended by its controller, while small divergence rate ($\lambda_{i2} > 1$) implies that the states of the open-loop system does not diverge greatly.

From (16), it is easy to verify the inequality (22) is guaranteed if the following inequalities are satisfied:

$$\begin{aligned} &P_{i\alpha} \leq \mu P_{i\beta},\ Q_{1i\alpha} \leq \mu Q_{1i\beta},\ Q_{2i\alpha} \leq \mu Q_{2i\beta}, \\ &Q_{3i\alpha} \leq \mu Q_{3i\beta},\ Z_{1i\alpha} \leq \mu Z_{1i\beta},\ Z_{2i\alpha} \leq \mu Z_{2i\beta}, \\ &\alpha, \beta \in \{1,2\},\ i = 1,2,\ldots,N \end{aligned} \tag{27}$$

Applying Lemmas 1 and 2, thus, we obtain already simultaneous stability conditions for the collection of NCSs. In Section III, the subscript i has been dropped for the sake of brevity, while it will be picked up in the following theorem for clarity.

Theorem 1: Under Assumptions 1-2, consider the collection of NCSs (1) and (2) with communication constraints as shown in Fig. 1. For given controller gains K_i, a integer $0 < C_{\max} < N$, scalars $\gamma_{ij} > 0$, $0 < \lambda_{i1} < 1$, $\lambda_{i2} > \lambda_{i1}$, $\overline{d}_i \geq \underline{d}_i \geq 0$, $\mu_i \geq 1$, if there exist positive-definite matrices $P_{ij} > 0$, $Q_{1ij} \geq 0$, $Q_{2ij} \geq 0$, $Q_{3ij} \geq 0$, $Z_{1ij} > 0$, $Z_{2ij} > 0$, and appropriately dimensioned matrices V_{ijq}, W_{ijq}, S_{ijq}, $i = 1,2,\ldots,N$, $j \in \{1,2\}$, such that inequalities (17) in Lemma 1 and (27), and the schedulable condition (25) hold. Then, there exists a scheduling policy such that all N systems in (1) and (2) are exponentially stable with weighted L_2

gains $\sqrt{c_i}\gamma_{i0}$ ($c_i > 0$, $\gamma_{i0} = \max_{j\in\{1,2\}} \gamma_{ij}$) and decay rates $0 < \rho_i < 1$.

Remark 4: To obtain a large value of N, it is desirable that λ_{ij} are small according to Remark 3. Small average dwell time lower bound T_a^* is also desirable [20], [12], [15]. (17) and (27) are bilinear matrix inequality (BMI) because of the product of unknown scalars and matrices. The following method can be used to find λ_{ij} and μ_i:

Step 1: Choose sufficiently large initial λ_{ij}^0 and μ_i^0 such that there exists a feasible solution to (17) and (27).

Step 2: Set $\lambda_{ij} = \lambda_{ij}^0$, $\mu_i = \mu_i^0$ and solve LMIs (17) and (27).

Step 3: If there exists a feasible solution to (17) and (27), and then return to *Step 2* after decreasing λ_{ij}^0 and μ_i^0 to some extent. Otherwise, exist.

V. Example

Consider an unstable batch reactor (BR) [3] described by

$$\dot{x} = A^c x + B^c u \tag{28}$$

where parameters are obtained from [3]. The equivalent discrete-time model is described by

$$x(k+1) = Ax(k) + Bu(k) \tag{29}$$

where $A = e^{A^c h}$, $B = \int_0^h e^{A^c s} ds B^c$, and h is the sampling period. Assume $h = 0.05s$, network-induced delay $0 \le \tau(k) \le 1$, the maximum packet-dropout bound is $\bar{\xi} = 1$, and controller gain is given by

$$K = \begin{bmatrix} 0.3296 & -0.3977 & 0.1870 & -0.6036 \\ 2.5291 & 0.2441 & 1.7956 & -1.1687 \end{bmatrix}.$$

For simplicity, suppose that $C_{\max} = 1$, i.e, only one plant can close its feedback loop at any time instant. By Remark 4, we obtain $\lambda_1 = 0.73$, $\lambda_2 = 1.25$ and $\mu = 1.36$. Further, we have

$$\tilde{\lambda} = \frac{\ln\lambda_2}{\ln\lambda_2 - \ln\lambda_1} = 0.4149$$

which results in $2 * \tilde{\lambda} = 0.8298 < 1$ and $N = 2$ satisfying (25). Therefore, it can be concluded from Lemma 3 that such two identical unstable batch reactors can share a common commination network. It should be mentioned that the two unstable batch reactors can also share a common controller because they have the same dynamics.

Assuming the desired decay rate $\rho = 0.998$ and using the scheduling policy in Lemma 3, we have the following parameters: $\varepsilon = \bar{\varepsilon} = 0.1702$, $\lambda^* = 0.9627$, $\beta = 0.4855$, $\mathcal{T} = 32.3939$, $\lceil \beta_i \mathcal{T} \rceil = 17$. Label one of the two batch reactors as BR1, and the other as BR2. Therefore, the static period scheduling policy is determined: the scheduling is cyclic with closing the control loop: BR1 $\to$ BR2 $\to$ BR1, and each duration on the batch reactor is 17 time steps. Hence the time period of the scheduling policy is 34 time steps.

Let the initial states $x_1(0) = x_2(0) = [1, 0, 0, 0]$. Under the above period scheduling policy, the state trajectories of the discrete-time model of BR1 and BR2 are shown in Figs. 2 and 3, respectively, where the network-induce delay and the data packet dropout are generated randomly.

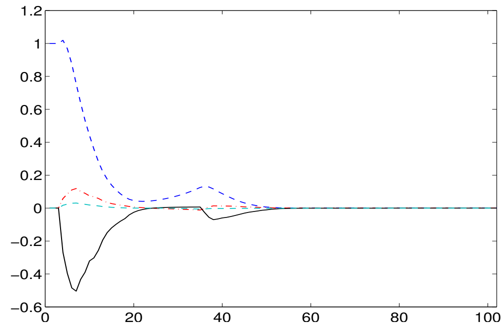

Fig. 2. State trajectories of BR1.

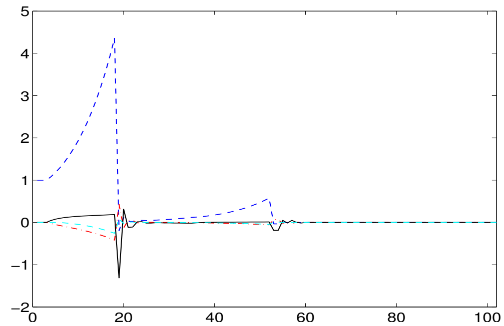

Fig. 3. State trajectories of BR2.

VI. Conclusions

In this paper, a switched delay system approach has been applied in the study of simultaneous stabilization with L_2 gain performances for a collection of NCSs under communication constraints. Sufficient stability conditions have been presented for an individual plant whose feedback control loop is closed/open via a shared unreliable communication network. Simultaneous stability of the collection of NCSs has been proposed under a static periodic scheduling policy. Future work will be devoted to jointly design feedback control and scheduling policy for the NCSs.

Appendix A: Proof of Lemma 2

Without loss of generality, we assume that the plant is closed-loop during $[k_{2l}\ k_{2l+1})$, and its control loop is open during $[k_{2l+1}\ k_{2l+2})$, $l = 0, 1, \cdots$, where $k_0 = 0$.

i) If $k \in [k_{2l+1}\ k_{2l+2})$, it follows from (20), (22), and Definition 1 that

$$\begin{aligned} V(k) &\le \mu d_\lambda \lambda_2^{k-k_{2l+1}} \lambda_1^{k_{2l+1}-k_{2l}} V_2(k_{2l}) \\ &\quad -\mu d_\lambda \lambda_2^{k-k_{2l+1}} \sum_{s=k_{2l}}^{k_{2l+1}-1} \lambda_1^{(k_{2l+1}-s-1)} \Gamma_0(s) \\ &\quad - \sum_{s=k_{2l+1}}^{k-1} \lambda_2^{(k-s-1)} \Gamma_0(s) \\ &\le \cdots \\ &\le \mu^{2N(k)} d_\lambda^{N(k)} \lambda_1^{\alpha_c(k)} \lambda_2^{(k-\alpha_c(k))} V_1(0) \\ &\quad - \sum_{s=0}^{k-1} \mu^{2(N(k)-N(s))} d_\lambda^{(N(k)-N(s))} \\ &\quad \lambda_1^{\alpha_c(k)-\alpha_c(s)} \lambda_2^{(k-\alpha_c(k)+\alpha_c(s)-s-1)} \Gamma_0(s) \end{aligned} \tag{30}$$

where $\Gamma_0(k) = z^T(k)z(k) - \gamma_0^2\omega^T(k)\omega(k)$, and $\lambda_2 > \lambda_1$ and $N(k) = l+1$ are used.

ii) According to the Definition 1, it is clear that $N(k) = l$ for $k \in [k_{2l}\ k_{2l+1})$. Following the discussion on $k \in [k_{2l+1}\ k_{2l+2})$, if $k \in [k_{2l}\ k_{2l+1})$ we also have $V(k)$ is not more than the last term in (30).

Based on the above discussions, we can conclude that $V(k)$ is not more than the last term in (30) for any $k > 0$. From (23), we have

$$(\ln\lambda_2 - \ln\lambda_1)\alpha_c(k) \geq (\ln\lambda_2 - \ln\lambda^*)k$$

which is equivalent to

$$\lambda_1^{\alpha_c(k)}\lambda_2^{k-\alpha_c(k)} \leq (\lambda^*)^k \tag{31}$$

From (24), we have

$$\begin{aligned}\mu^{2N(k)}d_\lambda^{N(k)} &\leq e^{N_0(2\ln\mu+\ln d_\lambda)}e^{(k/T_a)(2\ln\mu+\ln d_\lambda)} \\ &\leq ce^{k(2\ln\rho+\ln\lambda^*)} = c(\frac{\rho^2}{\lambda^*})^k \end{aligned}\tag{32}$$

When $\omega(k) = 0$ and $\Gamma_j(s) = 0$, combining (30), (31) and (32) yields

$$V(k) \leq c\rho^{2k}V(0) \tag{33}$$

From the piecewise Lyapunov-like function (21) and (33), it follows that there exist constant $a > 0$ and $b > 0$ such that

$$a\|x(k)\|^2 \leq V(k),\ V(0) \leq b\|x(0)\|_\delta^2$$

which means that system (12) is robustly exponentially stable with decay rate $0 < \rho < 1$ when $\omega(k) = 0$, where $\|\cdot\|$ denotes the Euclidean norm and $\|x(0)\|_\delta = \sup_{-\overline{d}\leq\delta\leq 0}\{\|x(\delta)\|\}$.

Under the zero initial condition, system (12) with weighted L_2 gain can be verified as follows. For $x(k) = 0$ and $V(k) \geq 0$, it follows from (30) that

$$\begin{aligned}&\sum_{s=0}^{k-1}\mu^{2(N(k)-N(s))}d_\lambda^{(N(k)-N(s))}\lambda_1^{\alpha_c(k)-\alpha_c(s)} \\ &\quad\times\lambda_2^{(k-\alpha_c(k)+\alpha_c(s)-s-1)}\Gamma_0(s) \leq 0\end{aligned}\tag{34}$$

Multiplying both sides of (34) by $\mu^{-2N(k)}d_\lambda^{-N(k)}\lambda_1^{-\alpha_c(k)}$ $\lambda_2^{-(k-\alpha_c(k)-1)}$ yields

$$\sum_{s=0}^{k-1}\mu^{-2N(s)}d_\lambda^{-N(s)}(\lambda^*)^{-s}\Gamma_0(s) \leq 0 \tag{35}$$

where the inequality $(\lambda^*)^{-s} \leq \lambda_1^{-\alpha_c(s)}\lambda_2^{-s+\alpha_c(s)}$ is used according to (31). Considering $\mu \geq 1$, $d_\lambda \geq 1$, and (32), we have

$$c^{-1}(\frac{\rho^2}{\lambda^*})^{-s} \leq \mu^{-2N(s)}d_\lambda^{-N(s)} \leq 1 \tag{36}$$

Combining (35) and (36) gives

$$\sum_{s=0}^{k-1}(\frac{\lambda^*}{\rho^2})^s z^T(s)z(s) \leq c\gamma_0^2\sum_{s=0}^{k-1}\omega^T(s)\omega(s) \tag{37}$$

which means that system (12) with weighted L_2 gain $\sqrt{c}\gamma_0$ according to the definition of weighted L_2 gain in Definition 3 [22], where $0 < \lambda^*/\rho^2 < 1$.

REFERENCES

[1] W. Ren, H. Chao, W. Bourgeous, N. Sorensen, and Y. Q. Chen, "Experimental validation of consensus algorithms for multivehicle cooperative control," *IEEE Transactions on Control Systems Technology*, vol. 16, no. 4, pp. 745–752, 2008.

[2] J. Finke, K. M. Passino, and A. G. Sparks, "Stable task load balancing strategies for cooperative control of networked autonomous air vehicles," *IEEE Transactions on Control Systems Technology*, vol. 14, no. 5, pp. 789–803, 2006.

[3] G. C. Walsh, H. Ye, and L. G. Bushnell, "Stability analysis of networked control systems," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 3, pp. 438–446, 2002.

[4] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, 2007.

[5] D. Hristu-Varsakelis, "Feedback control systems as users of a shared network: communication sequences that guarantee stability," in *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, Florida USA, 2001, pp. 3631–3636.

[6] ——, "Feedback control with communication constraints," in *Handbook of Networked and Embedded Control Systems*, Boston, MA: Birkhäuser, 2005, pp. 575–599.

[7] D. Nešić and A. R. Teel, "Input-output stability properties of networked control systems," *IEEE Transactions on Automatic Control*, vol. 49, no. 10, pp. 1650–1667, 2004.

[8] D. Hristu-Varsakelis, "Interrupt-based feedback control over shared commmnication medium," in *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, Nevada USA, 2002, pp. 3233–3228.

[9] M. S. Branicky, S. M. Phillips, and W. Zhang, "Scheduling and feedback co-design for networked control systems," in *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, Nevada USA, 2002, pp. 1211–1217.

[10] H. Lin, G. Zhai, L. Fang, and P. J. Antsaklis, "Stability and H_∞ performace preserving scheduling policy for networked control systems," in *Proceedings of the 16th IFAC World Congress*, Prague, Czech Republic, 2005.

[11] S.-L. Dai, H. Lin, and S. S. Ge, "Scheduling and control co-design for a collection of networked control systems with uncertain delays," *To apprear in IEEE Transactions on Control Systems Technology*, 2009.

[12] D. Liberzon, *Switching in Systems and Control*. MA: Birkhäuser: Boston, 2003.

[13] Z. Sun and S. S. Ge, *Switched Linear Systems–Control and Design*. New York: Springer-Verlag, 2005.

[14] H. Lin and P. J. Antsaklis, "Stability and stabilizability of switched linear systems: A survey of recent results," *IEEE Transactions on Automatic Control*, vol. 54, no. 2, pp. 308–322, 2009.

[15] G. Zhai and H. Lin, "Controller failure time analysis for symmetric H_∞ control systems," *International Journal of Control*, vol. 77, no. 6, pp. 598–605, 2004.

[16] Y. G. Sun, L. Wang, and G. Xie, "Delay-dependent robust stability and H_∞ control for uncertain discrete-time switched systems with mode-dependent time delays," *Applied Mathematics and Computation*, vol. 187, pp. 1228–1237, 2007.

[17] D. Du, B. Jiang, P. Shi, and S. Zhou, "H_∞ filtering of discrete-time switched systems with state delays via switched Lyapunov function approach," *IEEE Transactions on Automatic Control*, vol. 52, no. 8, pp. 1520–1525, 2007.

[18] L. Zhang, P. Shi, and E. H. Boukas, "H_∞ output-feedback control for switched linear discrete-timesystems with time-varying delays," *International Journal of Control*, vol. 80, no. 8, pp. 1354–1365, 2007.

[19] X.-M. Sun, J. Zhao, and W. Wang, "State feedback control for discrete delay systems with controller failures based on average dwell-time method," *IET Control Theory and Applications*, vol. 2, no. 2, pp. 126–132, 2008.

[20] J. P. Hespanha and A. S. Morse, "Stability of switched systems with average dwell-time," in *Proceedings of the 38th IEEE Conference on Decision and Control*, Phoenix, Arizona USA, 1999, pp. 2655–2660.

[21] Y. He, M. Wu, G.-P. Liu, and J.-H. She, "Output feedback stabilization for a discrete-time system with a time-varying delay," *IEEE Transactions on Automatic Control*, vol. 53, no. 10, pp. 2373–2377, 2008.

[22] L. Zhang, E. K. Boukas, and P. Shi, "Exponential H_∞ filtering for uncertain discrete-time switched linear systems with average dwell time: A μ-dependent approach," *International Journal of Robust and Nonlinear Control*, vol. 18, no. 11, 2008.