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Simultaneous Stability of a Collection of Networked Control Systems with Uncertain Delays

Shi-Lu Dai[a,b], Hai Lin[a], Shuzhi Sam Ge[a], Xiaoyang Li[a]
[a]Department of Electrical & Computer Engineering, National University of Singapore, Singapore 117576
[b]Key Laboratory of Integrated Automation of Process Industry, Ministry of Education of China,
Northeastern University, Shenyang, P.R. China 110004
E-mail addresses: {shiludai; elelh; samge; xiaoyang_li}@nus.edu.sg

***Abstract*—This paper is concerned with simultaneous stability of a collection of continuous-time linear plants whose feedback control loops are closed via a shared digital communication network. Because of the limitation of communication capacity, only a limited number of controller-plant connections can be accommodated at any time instant. Therefore, it is necessary to carefully design the scheduling policy so as to achieve simultaneous stabilization for all these control loops. In the paper, sufficient condition on the existence of such a scheduling policy is presented for the collection of networked LTI systems with sampled-data controllers and network-induced uncertain delays. It turns out that the condition is only based on the convergence rate of the closed-loop system and the divergence rate of the open-loop plant. The proof for this schedulability condition is in a constructive way, which can also serve as a systematic way for the scheduling policy design.**

***Index Terms*—Networked control systems (NCSs), hybrid systems, scheduling, communication constraints, average dwell time.**

I. Introduction

Networked control systems (NCSs) are feedback control systems in which the communication between spatially distributed system components like sensors, actuators and controllers occurs through shared band-limited digital communication networks. Compared with conventional point-to-point interconnected control systems, NCSs possess many attractive features due to the inclusion of a communication network. These advantages are higher system testability and resource utilization, as well as lower cost, reduced weight and power, simpler installation and maintenance [1], [2], which make the use of network in control systems connecting sensors/actuators to controllers more and more popular in many applications, including traffic control, satellite clusters, mobile robotics [3]. Consequently, considerable attention has been paid to the study of NCSs recently, see for example the survey papers [3]–[5], the recent special issue [6], and references therein.

In NCSs, it is common that many spatially distributed system components, like sensors, controllers and actuators, share a common communication network. In this paper, we consider a class of NCSs consisting of a collection of continuous-time linear time-invariant (LTI) plants whose open-loop dynamics are unstable. Each plant communicates with its remotely located controller over a shared network link, as depicted in Fig. 1. Because of limited communication capacities, controller-plant communication is restricted and not all the control loops in the

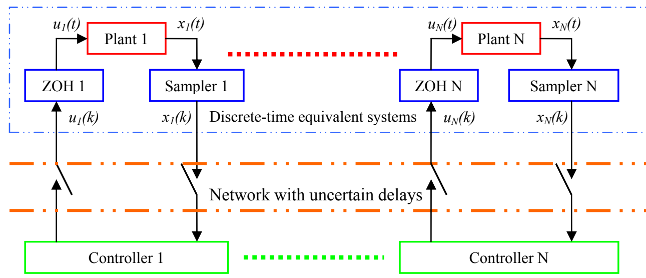

Fig. 1. Communication of controller-plants through a shared network link

NCSs can be addressed at the same time. Suppose that only a few controller-plant connections can be granted at any one time, while the other feedback control loops are assumed to be open-loop. It is clear that if some connections monopolize the network, then other plants will not be stabilizable. In order to guarantee stabilization of each plant, it is necessary to design a scheduling algorithm to schedule the NCSs. It should be pointed out that similar problems have been studied in [7]–[10]. In [7], the rate-monotonic scheduling algorithm was applied to schedule a set of NCSs. A schedulability condition was presented in [8], [9] for simultaneous stability of a group of continuous-time linear system by using a common Lyapunov function, and a time-division based scheduling policy was developed in [10] by employing average dwell time technique incorporated with piecewise Lyapunov-like functions. So far, all these studies are carried out in continuous-time domain and under the assumption that no time delay happens. We are going to study the problem in a sampled-data control framework and explicitly consider uncertain network-induced delays in the control-loops. The motivation comes from the popularity of digital control and unavoidable transmission delays in communication networks. Delays can usually degrade system's performance and even cause instability. In view of discretizing continuous-time linear systems affected by time-varying delays, these systems can be described by linear systems with polytopic uncertainties, and then robust control methods are applied for these uncertain systems [11]–[13].

In this paper, we address the simultaneous stability problem for the collection of networked LTI system. The main contributions of this paper lies in that: i) parameter-dependent Lyapunov function method combined with average dwell time is applied to derive stability conditions for a single control loop; and ii) a schedulability condition is presented by employing a constructive proof method, which provides a systematic way to design a scheduling policy. The rest of the paper is organized as follows. In Section II, the NCSs under consideration are modeled as discrete-time polytopic uncertain systems with one step delay. Simultaneous stability conditions are proposed for the collection of systems in Section III. An illustrative example is provided in Section IV to demonstrate the effectiveness of the proposed method. Finally, conclusions and future works are included in Section V.

II. Problem formulation and preliminaries

Consider the NCSs consisting of a collection of continuous-time LTI plants whose feedback control loops are closed via a shared network link, as illustrated in Fig. 1, where the *i*-th plant $i = 1, ..., N$ is given by

$$\dot{x}_i(t) = A_i^c x_i(t) + B_i^c u_i(t) \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the system state, $u_i(t) \in \mathbb{R}^{m_i}$ is the control input. The pair $(A_i^c \ B_i^c)$ is assumed to be controllable, but A_i^c is unstable.

Assumption 1: [8], [9] Due to a limited communication capacity, not all the control loops in the NCSs can be addressed at the same time. Assumed that a maximum of $C_{max} < N$ plants may close their feedback loops at any one time, while the other control loops are assumed to be open-loop.

Assumption 2: [8], [9] When a plant fails to communicate with its corresponding controller, the open-loop system is unstable; otherwise, the plant may communicate with its controller and the resulting closed-loop system is stable.

Assumption 3: [13] The following setup is considered in this paper: a clock-driven sensor, that periodically samples the plant outputs; an event-driven controller, which calculates the control signal as soon as the sensor data arrive; an event-driven actuator, that updates the plant inputs as soon as the controller data arrive.

Assumption 4: [13] Consider transmission delay in a data network and suppose that network-induced delays $\tau_i(k)$ are unknown time-varying and satisfy

$$0 \leq d_{i1} \leq \tau_i(k) \leq d_{i2} < h_i \tag{2}$$

where d_{i1} and d_{i2} are constant positive scalars representing the minimum and maximum delays, respectively.

Sampling system (1) with period h_i, we obtain the following discrete representation [14]:

$$x_i(k+1) = \Phi_i x_i(k) + \Gamma_i^0(\tau_i(k))u_i(k) + \Gamma_i^1(\tau_i(k))u_i(k-1) \tag{3}$$

where $\Phi_i = e^{A_i^c h_i}$, $k \in \mathbb{Z}^+$ (nonnegative integers), and

$$\Gamma_i^0(\tau_i(k)) = \int_0^{h_i-\tau_i(k)} e^{A_i^c s} ds B_i^c \tag{4}$$

$$\Gamma_i^1(\tau_i(k)) = \int_{h_i-\tau_i(k)}^{h_i} e^{A_i^c s} ds B_i^c = \Gamma_i - \Gamma_i^0(\tau_i(k)) \tag{5}$$

with $\Gamma_i = \int_0^{h_i} e^{A_i^c s} ds B_i^c$. Note that $\Gamma_i^0(\tau_i(k))$ and $\Gamma_i^1(\tau_i(k))$ are dependent on the unknown time-varying delay $\tau_i(k)$ and system (3) is an uncertain linear system with time-varying uncertainty. Since $\Gamma_i^0(\tau_i(k))$ cannot take any value, it belongs to the convex hull [11]–[13]:

$$\Gamma_i^0(\tau_i(k)) \in \mathrm{co}\{G_{i1}, \ldots, G_{ir}\}. \tag{6}$$

Remark 1: There exist some approaches which can be used to obtain vertices of the convex hull (6) in the literature, see [11], [12] for example.

Consider system (3) with (4)-(6), and the static state-feedback law $u_i(k) = K_i x(k)$, thus the closed-loop system is given by

$$x_i(k+1) = \mathcal{A}_{ci} x_i(k) + \mathcal{B}_{ci} x_i(k-1) \tag{7}$$

where

$$\begin{aligned}
&\mathcal{A}_{ci} = \sum_{j=1}^{r} \theta_{ij}(k) A_{cij},\ \mathcal{B}_{ci} = \sum_{j=1}^{r} \theta_{ij}(k) B_{cij},\\
&\sum_{j=1}^{r} \theta_{ij} = 1,\ \theta_{ij}(k) \geq 0, A_{cij} = \Phi_i + G_{ij} K_i,\\
&B_{cij} = \Gamma_i K_i - G_{ij} K_i.
\end{aligned}$$

It is clear that system (7) is a discrete-time polytopic uncertain system with one step delay.

Let $u_i(k) = K_i x_i(k)$ be state-feedback law rendering the *i*-th closed-loop system exponentially stable. The objective of this paper is to determine a scheduling policy such that all systems in (7) are exponentially stabilized.

III. Stability for NCSs

A. Single control loop stability analysis

In this subsection, the subscript *i* will be dropped for short. Without causing confusion, we use $[0 \ k)$ to denote the time interval $[0 \ kh_i)$, where h_i is the sampling period of the *i*-th plant. Throughout the paper, the subscripts "*c*" and "*o*" stand for closed-loop and open-loop systems, respectively.

This subsection focus on the stability of a single plant with closed/open feedback control loops. It is supposed that the single control system is open-loop for some time because the shared network link is occupied by another network user. The single control system can be described by a switched system, which is composed of the open-loop subsystem

$$x(k+1) = A_o x(k) \tag{8}$$

and the closed-loop subsystem

$$x(k+1) = \mathcal{A}_c x(k) + \mathcal{B}_c x(k-1) \tag{9}$$

where $A_o = \Phi = e^{A^c h}$, $\mathcal{A}_c$ and $\mathcal{B}_c$ are defined in (7).

Definition 1: The system $x(k+1) = f(x(k))$ with $f(0) = 0$ is said to be exponentially stable with decay rate $0 < \rho < 1$ if $\| x(k) \| \leq c\rho^{k-k_0} \| x(0) \|$, $\forall k \geq k_0$ holds for a constant $c > 0$.

Definition 2: [10] For any $k > 0$, we denote by $\alpha_{ci}(k)$ the total number of sampling periods of the i-th plant being closed-loop (attended by the controller) during $[0\ k)$, and call the ratio $\frac{\alpha_{ci}(k)}{k}$ the attention rate of the i-th plant. $N_i(k)$ denotes the total number of switching for the i-th plant between open-loop and closed-loop status, thus $N_i(k)$ is said to be the attention frequency.

For the single control system composed of subsystem (8) and subsystem (9), choose the following piecewise Lyapunov-like function candidate as

$$V(k) = \begin{cases} V_c(k), & \text{if closed-loop} \\ V_o(k), & \text{if open-loop.} \end{cases} \tag{10}$$

The following result gives exponential stability of a single control system.

Lemma 1: Consider the single control system composed of the unstable open-loop subsystem (8) and the stable closed-loop subsystem (9). The single control system is exponentially stable with decay rate $0 < \rho < 1$ if the following conditions hold:

i) The positive functions $V_c(k)$ and $V_o(k)$ in (10) satisfy

$$V_c(k+1) \leq \lambda_c V_c(k), \quad V_o(k+1) \leq \lambda_o V_o(k) \tag{11}$$

where $0 < \lambda_c < 1$ and $\lambda_o > 1$;

ii) there exists a constant scalar $\mu > 1$ such that

$$V_c(k) \leq \mu V_o(k), \quad V_o(k) \leq \mu V_c(k) \tag{12}$$

for any $x(k)$;

iii) the attention rate satisfies

$$\frac{\alpha_c(k)}{k} \geq \frac{\ln\lambda_o - \ln\lambda^*}{\ln\lambda_o - \ln\lambda_c} \tag{13}$$

iv) the attention frequency satisfies

$$\begin{aligned} N(k) &\leq N_0 + k/T_a,\ N_0 = \frac{\ln c}{\ln\mu}, \\ T_a &> T_a^* = \frac{\ln\mu}{2\ln\rho - \ln\lambda^*} \end{aligned} \tag{14}$$

where $\lambda_c < \lambda^* < \rho^2 < 1$, $c > 0$, T_a and N_0 are called average dwell time and the chatter bound, respectively.

Proof: See Appendix A.

Remark 2: Condition i) of Lemma 1 implies that the positive function $V_c(k)$ in (10) along the state trajectory of the closed-loop subsystem (9) has an exponential decay property: $V_c(k) \leq \lambda_c^{(k-k_0)} V_c(k_0)$, where $0 < \lambda_c < 1$ and k_0 is the initial time step. Moreover, $V_o(k+1) \leq \lambda_o V_o(k)$ means that $V_o(k) \leq \lambda_o^{(k-k_0)} V_c(k_o)$ ($\lambda_o > 1$), which gives an exponential increase of $V_o(k)$ along the state trajectory of the open-loop subsystem (8). Condition ii) first appeared in [15] and has almost become a standard in applying the average dwell time method to design switching laws for hybrid systems. To ensure the exponential stability of the system, condition iii) implies that the attention rate of the i-th plant is required to be sufficiently large, while the attention frequency is restricted with condition iv).

B. Scheduling policy for NCSs

In this subsection, we will concentrate on finding a scheduling policy for establishing and terminating communication between each system and its controller in a way that stabilizes all systems.

Lemma 2: Consider the NCSs with a common shared network (Fig. 1). Suppose that Conditions i) and ii) in Lemma 1 hold for any single control system, and the following condition holds:

$$\sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} < C_{max} \tag{15}$$

where $0 < \lambda_{ci} < 1$, $\lambda_{oi} > 1$, $C_{max} < N$, and C_{max} denotes the maximum of plants may communicate with their remote controllers at any one time. Then there exists a scheduling policy which guarantees exponential stabilization of each system (7).

Proof: See Appendix B.

Remark 3: The form of schedulability condition (15) is similar to the continuous-time one in [8]–[10]. The proof of Lemma 2 is a constructive, which provides a systematic way to design a scheduling policy such that all N systems are stabilized.

C. Simultaneous stability

In this subsection, we will give simultaneous stability conditions for the collection of networked LTI systems.

For the polytopic system (7) with closed/opened control loops, we choose the following piecewise parameter-dependant Lyapunov-like function candidate as

$$V_i(k) = \begin{cases} V_{ci}(k), & \text{if closed-loop} \\ V_{oi}(k), & \text{if open-loop} \end{cases} \tag{16}$$

where

$$\begin{aligned} V_{ci}(k) &= x_i^T(k)\mathcal{P}_{ci}x_i(k) + x_i^T(k-1)\mathcal{Q}_{ci}x_i(k-1) \\ V_{oi}(k) &= x_i^T(k)P_{oi}x_i(k) + x_i^T(k-1)Q_{oi}x_i(k-1) \end{aligned}$$

with $\mathcal{P}_{ci} = \sum_{j=1}^{r} \theta_{ij}(k)P_{cij}$, $\mathcal{Q}_{ci} = \sum_{j=1}^{r} \theta_{ij}(k)Q_{cij}$, $P_{cij} > 0$, $Q_{cij} > 0$, $P_{oi} > 0$, $Q_{oi} > 0$.

Theorem 1: Consider the NCSs with a common shared network (Fig. 1) and let $u_i(k) = K_i x_i(k)$ be state-feedback law rendering the i-th closed-loop system exponentially stable. For given a positive integer $C_{max} > 0$ and constants $0 < \lambda_{ci} < 1$, $\lambda_{oi} > 1$, $\mu_i > 1$, suppose that there exist positive-definite matrices $P_{oi} > 0$, $Q_{oi} > 0$, $P_{cij} > 0$, $P_{cil} > 0$, $Q_{cij} > 0$, $Q_{cil} > 0$, and matrices T_{cil} ($i = 1, \ldots, N,\ j, l = 1, 2, \ldots, r$) such that

$$\begin{bmatrix} \Omega_{c1} & \Omega_{c2} & \Omega_{c3} \\ * & -\lambda_{ci}P_{cij} + Q_{cil} & 0 \\ * & * & -\lambda_{ci}Q_{cij} \end{bmatrix} < 0 \tag{17}$$

$$A_{oi}^T P_{oi} A_{oi} - \lambda_{oi}P_{oi} + Q_{oi} < 0 \tag{18}$$

$$\begin{aligned} &P_{oi} \leq \mu_i P_{cij}, P_{cij} \leq \mu_i P_{oi} \\ &Q_{oi} \leq \mu_i Q_{cij}, Q_{cij} \leq \mu_i Q_{oi} \end{aligned} \tag{19}$$

$$\sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} < C_{max} \tag{20}$$

where $\Omega_{c1} = P_{cil} - T_{cil}^T - T_{cil}$, $\Omega_{c2} = T_{cil}(A_{oi} + G_{ij}K_i)$, $\Omega_{c3} = T_{cil}(\Gamma_i K_i - G_{ij}K_i)$, and $*$ denotes the symmetric terms in a symmetric matrix. Then there exists a scheduling policy which guarantees exponential stabilization of each system (7), and for every system, the state decay estimation is given by

$$\|x_i(k)\| \leq \sqrt{\frac{b_i c_i}{a_i}} \rho_i^k \|x_i(0)\|$$

where decay rate $0 < \rho_i < 1$, constants $c_i > 0$, $a_i = \min\{\lambda_{\min}(P_{cij})\ (j = 1, 2, \ldots, r),\ \lambda_{\min}(P_{oi})\}$, $b_i = \max\{\lambda_{\max}(P_{cij})\ (j = 1, 2, \ldots, r),\ \lambda_{\max}(P_{oi})\}$, and $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a symmetric matrix, respectively.

Proof: See Appendix C.

Remark 4: In order to obtain small λ_{ci} and λ_{oi}, we needs to solve the optimization problem subjected to inequalities (17)-(20). This optimization problem is Bilinear Matrix Inequality (BMI) problem because of the product of unknown scalars and matrices. BMI problems are NP-hard and not computationally efficient. However, a few piratical algorithms for BMI problems exist in the literature, such as approximations, heuristics, brand-and-bound, and local search [16].

The following result shows that exponential stability of each system (7) implies simultaneous stability of the original collection of continuous-time linear plants.

Proposition 1: If conditions (17)-(20) in Theorem 1 hold, then each system (7) is exponential stabilized and its original continuous-time system (1) is asymptotically stable.

Proof: The proof can be obtained by following the similar procedure in [11] and is, thus, omitted for conciseness.

IV. Example

Consider the simplified model of the cart and inverted pendulum process taken from [17], which is described by

$$\dot{x}(t) \quad = \quad A^c x(t) + B^c u(t) \tag{21}$$

where $A^c = \begin{bmatrix} 0 & 1 \\ 9 & 0 \end{bmatrix}$, $B^c = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$.
The eigenvalues of matrix A^c are $eig(A^c) = \{-3; 3\}$, which means that A^c is an unstable matrix.

Suppose that transmission delay $\tau_i(k)$ satisfies (2). We apply the procedure proposed in [12] to calculate the vertices of the convex hull (6), which are given by

$$G_1 = -\tfrac{1}{18}\begin{bmatrix} \overline{\alpha}_1 + \overline{\alpha}_2 \\ 3(-\overline{\alpha}_1 + \overline{\alpha}_2) \end{bmatrix}, G_2 = -\tfrac{1}{18}\begin{bmatrix} \overline{\alpha}_1 + \underline{\alpha}_2 \\ 3(-\overline{\alpha}_1 + \underline{\alpha}_2) \end{bmatrix},$$
$$G_3 = -\tfrac{1}{18}\begin{bmatrix} \underline{\alpha}_1 + \overline{\alpha}_2 \\ 3(-\underline{\alpha}_1 + \overline{\alpha}_2) \end{bmatrix}, G_4 = -\tfrac{1}{18}\begin{bmatrix} \underline{\alpha}_1 + \underline{\alpha}_2 \\ 3(-\underline{\alpha}_1 + \underline{\alpha}_2) \end{bmatrix},$$

where $\overline{\alpha}_1 = e^{-3(h-d_2)} - 1$, $\underline{\alpha}_1 = e^{-3(h-d_1)} - 1$,
$\overline{\alpha}_2 = e^{3(h-d_1)} - 1$, $\underline{\alpha}_2 = e^{3(h-d_2)} - 1$.

Let state-feedback gain $K = [116.7863\ 22.5267]$ and the sampling period $h = 0.06$. Assume the lower delay bound of $\tau(k)$ is $d_1 = 0$, we are interested in the relationship between the minimum value of λ_c and the upper delay bound d_2. When the sampling period $h = 0.06$, Table I lists the minimum values of λ_c for different d_2 by solving (17). It can be seen from Table I that the value of λ_c grows as d_2 increases. Moreover, when $d_2 = 0$ (i.e., no networked-induced delay), λ_c approaches zero, and thus N tends to infinity from the schedulability condition (15). When $d_2 = 0.03$, (17) is unfeasible.

TABLE I
Calculate the minimum values of λ_c for different values of d_2 ($d_1 = 0$)

d_2	0.005	0.01	0.015	0.02	0.025
λ_c	0.4100	0.5690	0.6605	0.7005	0.9745

For given constants $\mu = 6.05$, $\lambda_o = 1.47$, it can be checked that there exists a feasible solution to (18) and (19).

For simplicity, assume from now on that $C_{max} = 1$, i.e, only one plant can close its feedback loop at any one time.

Substituting $\lambda_o = 1.47$ and $\lambda_c = 0.6605$ into (20) gives

$$\tilde{\lambda} = \frac{\ln\lambda_o}{\ln\lambda_o - \ln\lambda_c} = 0.4816,$$

which results in $2 * \tilde{\lambda} = 0.9631 < 1$ and $N = 2$ satisfying (20). Therefore, we can conclude from Theorem 1 that such two identical systems with network-induce delay $0 \leq \tau(k) \leq 0.015$ can share a common network link, which only takes care of one control loop at a time. It follows by analogous analysis that three alike control loops can be scheduled under the assumption that $0 \leq \tau(k) \leq 0.005$.

V. Conclusions and future works

In this paper, simultaneous stability conditions were proposed for a collection of plants whose feedback control loops are closed via a shared communication network. The presence of feedback-based communication constraints and the effect of networked-induced delays were taken into account in the communication network. Based on average dwell time technique, a sufficient schedulability condition was derived. The proof for this schedulability condition was in a constructive way, which can also serves as a systematic way for the scheduling policy design.

Future works will be devoted to jointly design feedback control and scheduling policy, and take the effect of packet-dropouts into account for the class of NCSs.

Appendixes

A: Proof of Lemma 1.

Without loss of generality, we assume that the controller works during $[k_{2j}\ k_{2j+1})$, and the plant is open-loop during $[k_{2j+1}\ k_{2j+2})$, $j = 0, 1, \cdots$, where $k_0 = 0$. Choose the piecewise Lyapunov function-like candidate as (10). For any $k > 0$, it holds from (11) that

$$V(k) \leq \begin{cases} \lambda_c^{k-k_{2j}} V_c(k_{2j}), & \text{if } k_{2j} \leq k < k_{2j+1} \\ \lambda_o^{k-k_{2j+1}} V_o(k_{2j+1}), & \text{if } k_{2j+1} \leq k < k_{2j+2} \end{cases} \tag{22}$$

where $0 < \lambda_c < 1$ and $\lambda_o > 1$.

Therefore, if $k \in [k_{2j+1}\ k_{2j+2})$, it follows from (12) and Definition 2 that

$$\begin{aligned} V(k) &\leq \lambda_o^{k-k_{2j+1}} V_o(k_{2j+1}) \\ &\leq \mu\lambda_o^{k-k_{2j+1}} V_c(k_{2j+1}^-) \\ &\leq \mu\lambda_o^{k-k_{2j+1}} \lambda_c^{k_{2j+1}-k_{2j}} V_c(k_{2j}) \\ &\leq \cdots \\ &\leq \mu^{N(k)} \lambda_c^{\alpha_c(k)} \lambda_o^{(k-\alpha_c(k))} V(0) \end{aligned} \tag{23}$$

where $N(k)$, $\alpha(k)$ are defined in Definition 2, and k_{2j+1}^- denotes the time instant that is immediately before k_{2j+1}.

Similarly, we have $V(k) \leq \mu^{N(k)} \lambda_c^{\alpha_c(k)} \lambda_o^{(k-\alpha_c(k))} V(0)$ for $k \in [k_{2j}\ k_{2j+1})$. From (13), we have

$$(\ln\lambda_o - \ln\lambda_c)\alpha_c(k) \geq (\ln\lambda_o - \ln\lambda^*)k$$

which is equivalent to

$$\lambda_c^{\alpha_c(k)} \lambda_o^{k-\alpha_c(k)} \leq (\lambda^*)^k. \tag{24}$$

From (14), we have

$$\begin{aligned} \mu^{N(k)} &\leq \mu^{N_0+(k/T_a)} = \mu^{N_0}\mu^{k/T_a} \\ &\leq \mu^{N_0}\mu^{\frac{k(2\ln\rho-\ln\lambda^*)}{\ln\mu}} = c(\frac{\rho^2}{\lambda^*})^k. \end{aligned} \tag{25}$$

Combining (23), (24) and (25) yields

$$V(k) \leq c\rho^{2k} V(0). \tag{26}$$

If a quadratic form is considered for the function (10), then there exist $a_c > 0$, $a_o > 0$, $b_c > 0$, and $b_o > 0$ such that

$$\begin{gathered} a_c\|x(k)\|^2 \leq V_c(k), \quad a_o\|x(k)\|^2 \leq V_o(k), \\ V_c(0) \leq b_c\|x(0)\|^2, \quad V_o(0) \leq b_o\|x(0)\|^2 \end{gathered}$$

hold, which implies

$$a\|x(k)\|^2 \leq V(k), \quad V(0) \leq b\|x(0)\|^2 \tag{27}$$

where $a = \min\{a_c, a_o\}$ and $b = \max\{b_c, b_o\}$.
From (26) and (27), we have

$$\|x(k)\| \leq \sqrt{\frac{bc}{a}}\rho^k\|x(0)\| \tag{28}$$

which means that the single control system is exponentially stable with decay rate $0 < \rho < 1$.

B: Proof of Lemma 2.

The proof is motivated by the continuous-time one in [10]. From (15), we can conclude that there exists a positive scalar $\overline{\varepsilon}$ such that the following inequality

$$\sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{0i} - \ln\lambda_{ci}} + \varepsilon C_{max} \leq C_{max} \tag{29}$$

holds for $0 < \varepsilon \leq \overline{\varepsilon} < 1$. Set $\overline{\varepsilon} = 1 - \frac{1}{C_{max}} \sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi}-\ln\lambda_{ci}}$, then we have

$$\begin{aligned} &\sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} + \sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}}\varepsilon \\ &< \sum_{i=1}^{N} \frac{\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} + \varepsilon C_{max} \leq C_{max}. \end{aligned} \tag{30}$$

Therefore, we obtain

$$\sum_{i=1}^{N} \frac{\ln\lambda_{oi} + \varepsilon\ln\lambda_{oi}}{\ln\lambda_{oi} - \ln\lambda_{ci}} = \sum_{i=1}^{N} \frac{\ln\lambda_{oi} - \ln\lambda_{oi}^{-\varepsilon}}{\ln\lambda_{oi} - \ln\lambda_{ci}} < C_{max} \tag{31}$$

which holds for all $0 < \varepsilon \leq \overline{\varepsilon} < 1$. Let $\lambda_i^* = \lambda_{oi}^{-\varepsilon}$ and $\rho_i^2 = \lambda_{oi}^{-\frac{\varepsilon}{2}}$. Note that $0 < \lambda_{ci} < 1 < \lambda_{oi}$ and (31), then it is easy to verify that $0 < \lambda_{ci} < \lambda_i^* < \rho_i^2 < 1$ for $i = 1, ..., N$. Let

$$\beta_i = \frac{\ln\lambda_{oi} - \ln\lambda_i^*}{\ln\lambda_{oi} - \ln\lambda_{ci}}, \quad i = 1, ..., N \tag{32}$$

then we have $0 < \beta_i < 1$ and $\sum_{i=1}^{N} \beta_i < C_{max}$.

Next, we propose a periodic scheduling policy which guarantees the asymptotic stability of the NCSs.

(i) Choose $\mathcal{T} = \max_i\{L_i h_i\}$, where L_i is a positive integer sufficiently large to satisfy the average dwell time condition for the i-th plant. For example, we may set $L_i = \lceil T_{ai}^* \rceil$, where T_{ai}^* is the lower bound of the average dwell time T_{ai}, and $\lceil\cdot\rceil$ denotes the upper integer bound.

(ii) Close C_{max} control loops for their plants at any time instant. Activate the control loops from 1 to N in order, and let the i-th control loop work for a time interval of length $\lceil \beta_i\mathcal{T}/h_i \rceil h_i$ for $i = 1, \ldots, N$.

Now, we show that under the above scheduling policy the NCSs are exponentially stable with decay rate ρ_i.

For any $t = kh_i > 0$, it can be written as $t = n\mathcal{T} + \Delta$, $0 \leq \Delta < \mathcal{T}$, where n is a nonnegative integer and Δ is a real number. For $i = 1$, the following two cases need to be considered:

(i) If $\Delta < \lceil \beta_i\mathcal{T}/h_i \rceil h_i$, then $\alpha_{ci}(k) = n\lceil \beta_i\mathcal{T}/h_i \rceil + \lceil \Delta/h_i \rceil$ and $N(k) = n$. Then, we obtain

$$\begin{aligned} \frac{\alpha_{ci}(k)h_i}{kh_i} &= \frac{n\lceil \beta_i\mathcal{T}/h_i \rceil h_i + \lceil \Delta/h_i \rceil h_i}{n\mathcal{T} + \Delta} \\ &\geq \frac{n\beta_i\mathcal{T} + \Delta}{n\mathcal{T} + \Delta} \geq \frac{n\beta_i\mathcal{T} + \beta_i\Delta}{n\mathcal{T} + \Delta} = \beta_i \end{aligned}$$

and $\frac{k}{T_a} = \frac{kh_i}{T_a h_i} \geq \frac{t}{\mathcal{T}} \geq n$. Thus, it follows that $N(k) \leq N_0 + \frac{k}{T_a}$.

(ii) If $\Delta \geq \lceil \beta_i\mathcal{T}/h_i \rceil h_i$, then $\alpha_{ci}(k) = n\lceil \beta_i\mathcal{T}/h_i \rceil + \lceil \beta_i\mathcal{T}/h_i \rceil$ and $N(k) = n + 1$.
Then, we have

$$\begin{aligned} \frac{\alpha_{ci}(k)h_i}{kh_i} &= \frac{n\lceil \beta_i\mathcal{T}/h_i \rceil h_i + \lceil \beta_i\mathcal{T}/h_i \rceil h_i}{n\mathcal{T} + \Delta} \\ &\geq \frac{n\beta_i\mathcal{T} + \beta_i\mathcal{T}}{n\mathcal{T} + \Delta} > \frac{n\beta_i\mathcal{T} + \beta_i\mathcal{T}}{n\mathcal{T} + \mathcal{T}} = \beta_i \end{aligned}$$

and $\frac{k}{T_a} = \frac{kh_i}{T_a h_i} \geq \frac{t}{\mathcal{T}} \geq n$. If set $N_0 \geq 1$ which yields $c > \mu$, then we have $N(k) = n + 1 \leq N_0 + \frac{k}{T_a}$.

Therefore, Conditions iii) and iv) in Lemma 1 are both satisfied when $i = 1$.

For $i > 1$, we may simply shift the initial time t_0 to the beginning of the first closed-loop sampling period of the i-th plant, and adjust the initial state x_0 correspondingly. Then it reduces to the case $i = 1$, and Conditions iii) and iv) in Lemma 1 are both satisfied for this shifted the i-th plant. It is straightforward to show that the exponential stability between the time-shifted (by a finite constant) control system and the original system. Therefore, all the systems are exponentially stable with specified decay rate under the scheduling policy. This completes the proof.

C: Proof of Theorem 1.

Without causing confusion, the subscript *i* will be dropped for short in this proof. From Lemma 2, we know that there exists a scheduling policy that exponentially stabilizes all N systems if Conditions i) and ii) in Lemma 1 and (15) hold. It is obvious that (20) is the same as (15) and we are to prove that Conditions i) and ii) in Lemma 1 hold if (17), (18), and (19) hold for any fixed i.

First of all, it can be verified that $V_c(k+1) \leq \lambda_c V_c(k)$ can be derived from (17).

Let $\mathcal{P}_c^+ = \sum_{j=1}^{r} \theta_j(k+1)P_{cj} = \sum_{l=1}^{r} \theta_l(k)P_{cl}$ and $\mathcal{Q}_c^+ = \sum_{j=1}^{r} \theta_j(k+1)Q_{cj} = \sum_{l=1}^{r} \theta_l(k)Q_{cl}$.
From (16), the forward difference for $V_c(k)$ along the state trajectory of system (9) is given by

$$V_c(k+1) - \lambda_c V_c(k) := \xi^T(k)\Pi_c(\theta)\xi(k) \tag{33}$$

where $\xi(k) = [x^T(k) \quad x^T(k-1)]^T$ and

$$\Pi_c(\theta) = \begin{bmatrix} \mathcal{A}_c^T\mathcal{P}_c^+\mathcal{A}_c + \mathcal{Q}_c^+ - \lambda_c\mathcal{P}_c & \mathcal{A}_c^T\mathcal{P}_c^+\mathcal{B}_c \\ * & -\lambda_c\mathcal{Q}_c + \mathcal{B}_c^T\mathcal{P}_c^+\mathcal{B}_c \end{bmatrix}.$$

On the other hand, note that (17) implies $P_{cl} - T_{cl}^T - T_{cl} < 0$, and T_{cl} is obviously nonsingular. Since $P_{cl} > 0$, we have $(T_{cl} - P_{cl})^T P_{cl}^{-1}(T_{cl} - P_{cl}) > 0$ which is equivalent to

$$P_{cl} - T_{cl}^T - T_{cl} > -T_{cl}^T P_{cl}^{-1} T_{cl}. \tag{34}$$

From (17) and (34), we have

$$\begin{bmatrix} -T_{cl}^T P_{cl}^{-1} T_{cl} & T_{cl}A_{cj} & T_{cl}B_{cj} \\ * & -\lambda_c P_{cj} + Q_{cl} & 0 \\ * & * & -\lambda_c Q_{cj} \end{bmatrix} < 0. \tag{35}$$

Let $J_{cl} = diag\{T_{cl}^{-1}P_{cl}, I, I\}$. Multiplying (35) by J_{cl}^T and J_{cl} on the left and the right, respectively, yields

$$\Xi_{cjl} := \begin{bmatrix} -P_{cl} & P_{cl}A_{cj} & P_{cl}B_{cj} \\ * & -\lambda_c P_{cj} + Q_{cl} & 0 \\ * & * & -\lambda_c Q_{cj} \end{bmatrix} < 0. \tag{36}$$

Therefore, (36) leads to

$$\sum_{l=1}^{r}\sum_{j=1}^{r} \Xi_{cjl} = \begin{bmatrix} -\mathcal{P}_c^+ & \mathcal{P}_c^+\mathcal{A}_c & \mathcal{P}_c^+\mathcal{B}_c \\ * & -\lambda_c\mathcal{P}_c + \mathcal{Q}_c^+ & 0 \\ * & * & -\lambda_c\mathcal{Q}_c \end{bmatrix} < 0. \tag{37}$$

Let $\mathcal{J}_c = \begin{bmatrix} \mathcal{A}_c^T & I & 0 \\ \mathcal{B}_c^T & 0 & I \end{bmatrix}$. Multiplying (37) by $\mathcal{J}_c$ on the left and by $\mathcal{J}_c^T$ on the right, respectively, gives rise to

$$\Pi_c(\theta) = \begin{bmatrix} \mathcal{A}_c^T\mathcal{P}_c^+\mathcal{A}_c + \mathcal{Q}_c^+ - \lambda_c\mathcal{P}_c & \mathcal{A}_c^T\mathcal{P}_c^+\mathcal{B}_c \\ * & -\lambda_c\mathcal{Q}_c + \mathcal{B}_c^T\mathcal{P}_c^+\mathcal{B}_c \end{bmatrix} < 0.$$

Therefore, we can conclude that $V_c(k+1) \leq \lambda_c V_c(k)$ if (17) holds.

Secondly, it is easy to verify that (18) results in $V_o(k+1) \leq \lambda_o V_o(k)$ from (16) and (18). Therefore, we can conclude that Conditions i) in Lemma 1 are satisfied if (17) and (18) hold.

Thirdly, it follows from (16) and (19) that

$$P_o \leq \mu\mathcal{P}_c, \ \mathcal{P}_c \leq \mu P_o, \ Q_o \leq \mu\mathcal{Q}_c, \ \mathcal{Q}_c \leq \mu Q_o$$

which implies $V_c(k) \leq \mu V_o(k)$ and $V_o(k) \leq \mu V_c(k)$.

Lastly, for the considered Lyapunov function (16), it follows from (27) and (28) that

$$\|x_i(k)\| \leq \sqrt{\frac{b_i c_i}{a_i}} \rho_i^k \|x_i(0)\|$$

which thus completes the proof.

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