EE 40458
Power Gain and Amplifier Design

10/31/2017

Motivation

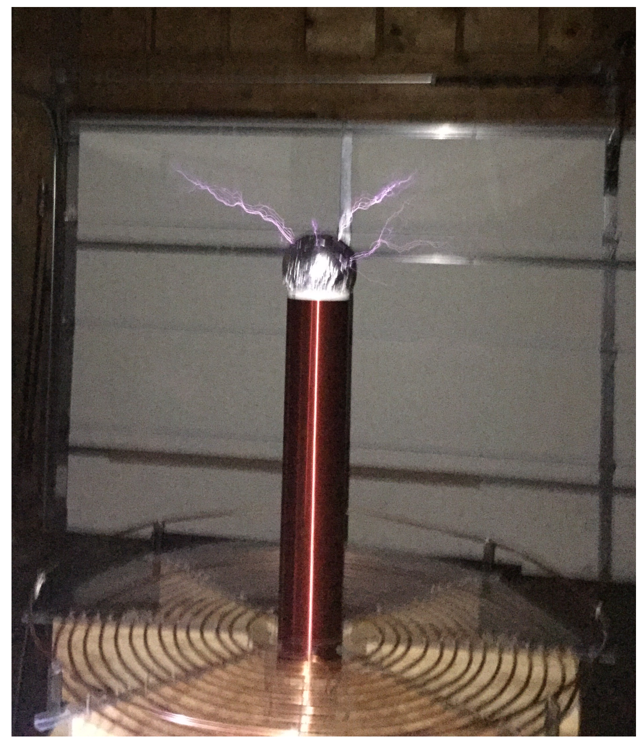
- Brief recap:
  - We’ve studied matching networks (several types, how to design them, bandwidth, how they work, etc…)
  - Studied network analysis techniques (matrices with S, ABCD, …; flow graphs, etc.)
  - But why? Ultimate goal: design a circuit that does something useful
- Amplifiers are a good starting point
  - Useful for many (all?) systems
  - Fundamental principles can be applied to other circuits

Design/Analysis Approach

- Could approach design many ways
  - "Regular" circuit design: use equivalent circuit model (hybrid-π, T model)
  - We'll take an alternative: treat active device as (known) 2-port S-matrix
- This approach allows us to be more general (results will apply more widely)
  - Example: we'll be able to project what the highest gain possible is, whether or not the circuit is stable (i.e., will it oscillate?); not always so obvious from equivalent circuits without a lot of work
  - Side bonus: we'll get to see what features of S-matrices are desirable for circuits; help us choose devices

Amplifiers – It's (Mostly) About Gain

- From previous classes—of course gain is the key thing
- But it isn't quite so simple for high-frequency systems
- Recall: from "regular" circuit design: gain was (mostly) voltage gain Av; sometimes (more rarely) current gain
  - But you don't really need an amplifier for either of these!
  - A transformer can produce voltage gain (and a lot of it…)
  - More generally, a matching network trades voltage gain for current gain (or visa-versa)
- What do amplifiers really do, then?
  - **Power gain** is the key feature: not just voltage or current in isolation



Power Gain

- Power gain: not quite so simple as it might look:

$$Gain = \frac{P_{out}}{P_{in}}$$

- OK—but what is P_{out} and P_{in}, exactly? There are several (useful) possibilities, we need to be clear which we mean
  - With voltage, current gain these ambiguities are less troublesome

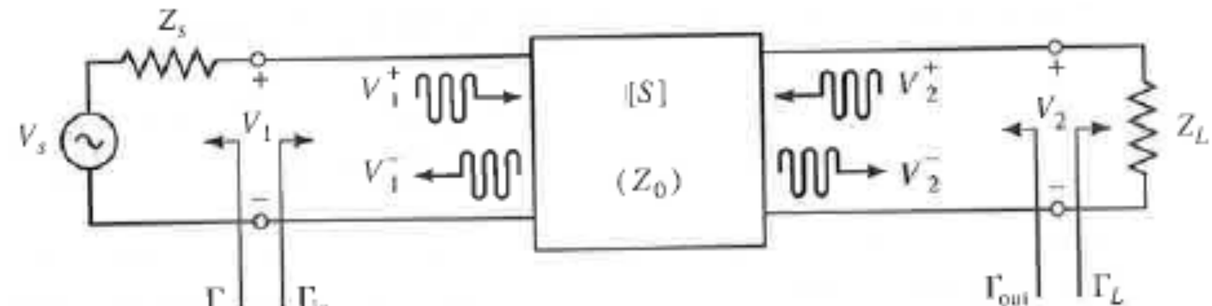
Power Gain – Definitions

- Since input, output power can be defined different ways, multiple definitions for power gain (not just one gain; there are several different alternatives!)

$$G_P = \frac{P_L}{P_{in}} = \frac{Power\ delivered\ to\ load}{Power\ delivered(absorbed\ by)\ network}$$

$$G_T = \frac{P_L}{P_{avs}} = \frac{Power\ delivered\ to\ load}{Power\ available\ from\ source}$$

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{Power\ available\ from\ network}{Power\ available\ from\ source}$$



Power Gain - Comparisons

- G_P: "operating power gain," most obvious, but not all that useful
- G_A: "available gain," good baseline for comparisons, shows what performance is possible

$$G_P = \frac{P_L}{P_{in}} = \frac{Power\ delivered\ to\ load}{Power\ delivered\,(absorbed\ by)\ network}$$

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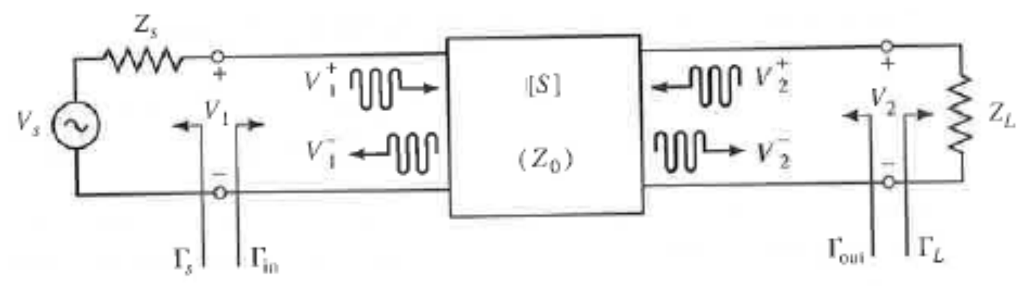
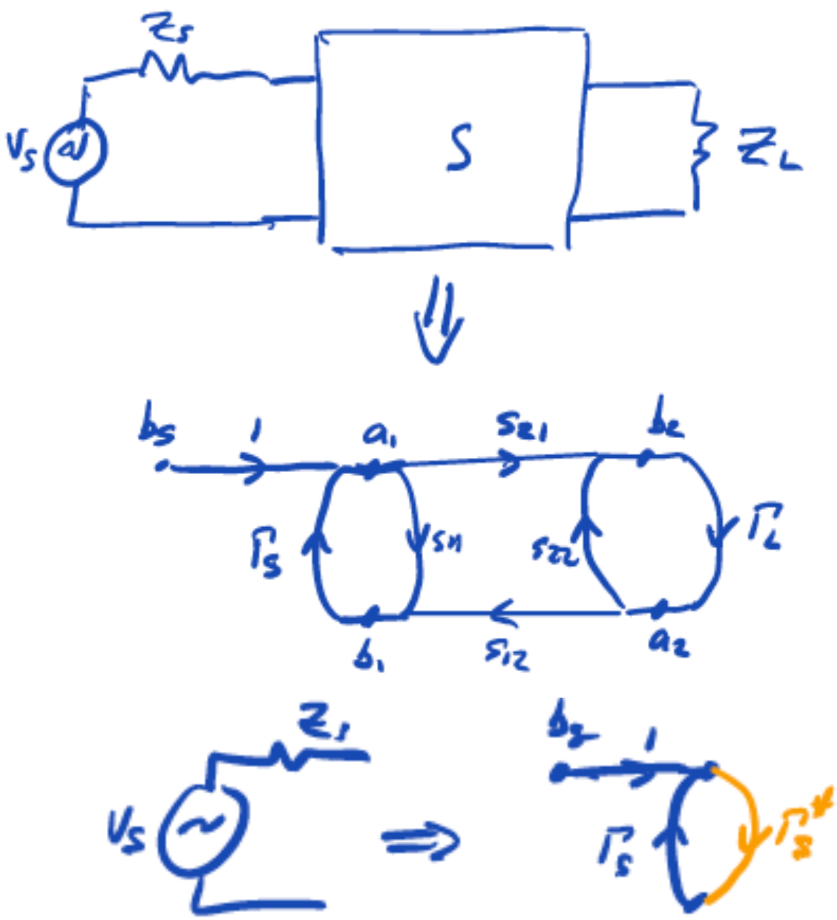
$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{Power\ available\ from\ network}{Power\ available\ from\ source}$$

- G_T: "transducer gain," most useful in system context, analysis
- What's the difference among these?
  - Do you count what actually gets into the network (P_{in}) or what could have gotten in (P_{avs})?
  - Same idea at output (P_L vs. P_{avs})
  - Degree of mismatch is the difference



Distinctions Among Power Gain Definitions

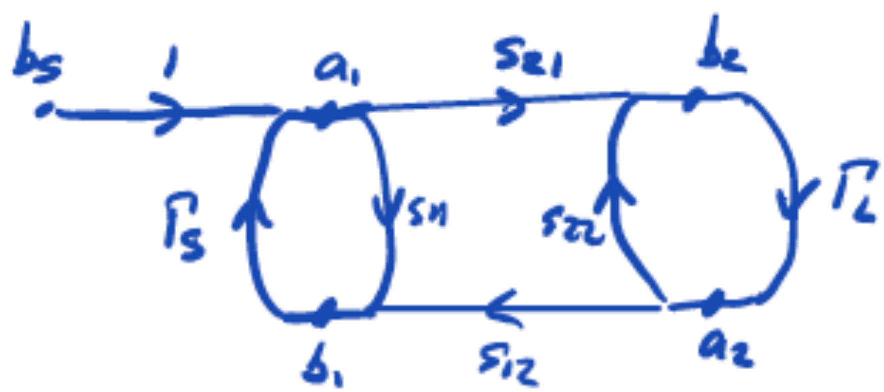
- How much power is transferred depends on the impedances/reflection coefficients:



- P_{in}: power carried by a_1, minus that reflected off (b_1)
- P_{avs}: what's the most power you could get from the source (if it were conjugately matched)?
- P_{avn}: what's the most power available from S?
- P_L: power dissipated in Z_L

Numerical Comparisons?

- How do these three definitions compare in terms of magnitude? $$G_T = \frac{P_L}{P_{avs}};\ \ G_P = \frac{P_L}{P_{in}};\ \ G_A = \frac{P_{avn}}{P_{avs}}$$
- $G_A > G_T$ (because $P_L < P_{avn}$ from mismatch)
- $G_P > G_T$ (because $P_{in} < P_{avs}$…)
- But not clear if G_P, G_A larger…depends on relative degree of source, load mismatch
- Can derive each gain from flow graph:



Power Gain Expressions

- Power gains:

$$G_T = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

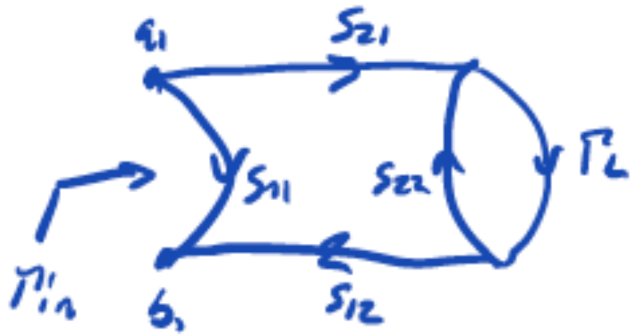
$$G_P = \frac{1}{1-|\Gamma_{in}|^2}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

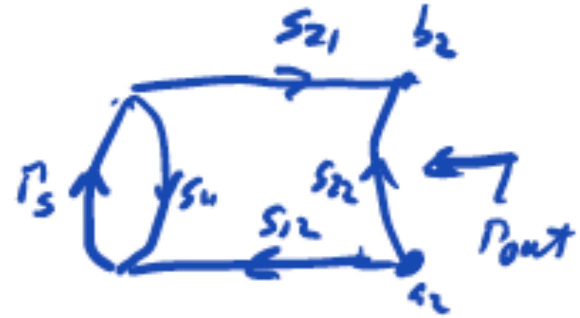
$$G_A = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2 \frac{1}{1-|\Gamma_{out}|^2}$$

- Where:

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$$





Amplifier Design

- With these definitions, we can start designing for real
- Several useful special cases for designs:
  - Maximum gain
  - Specified gain (i.e. trade lower gain for more bandwidth)
  - Low noise
- Let’s do an example of each one to see how they work

Design for Maximum Gain

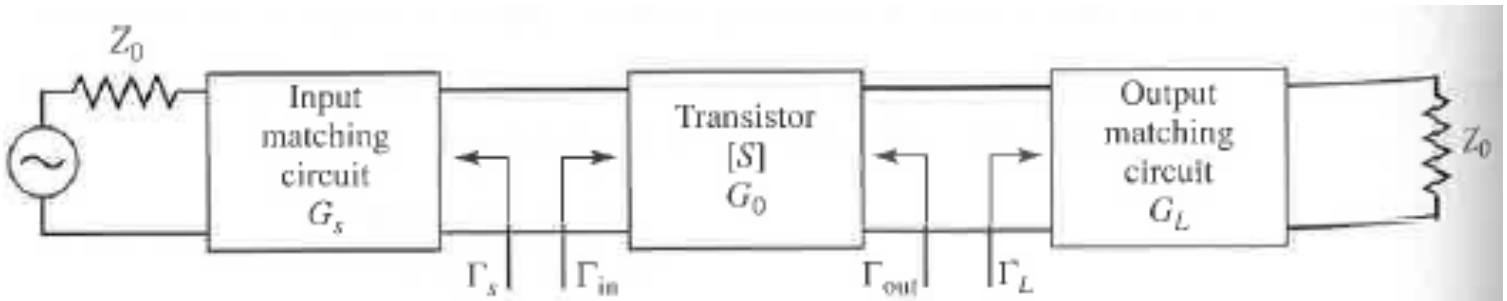
- For maximum gain: want to conjugately match both the input and output—simultaneously
- Mathematically: $\Gamma_{in} = \Gamma_S^*$

$$\Gamma_{out} = \Gamma_L^*$$

  - Note: can work with reflection coefficients since Z <-> Γ as a 1-to-1 map
  - Also, in this case, $G_T = G_A$; maximizing power in and out

Maximum Gain Design, Cont.

- So: how to design?
  - Basic idea:



  - Our job:
    - Make input matching network present $\Gamma_s^* = \Gamma_{in}$
    - Make output matching network present $\Gamma_L^* = \Gamma_{out}$
    - (So it is really just about making matching networks—once the correct "targets" for Γ_S, Γ_L are found)

Maximum Gain Design, Cont.

- Design approach: know input, output must be conjugately matched. So:

$$\Gamma_{in} = \Gamma_S^*$$

$$\Gamma_{out} = \Gamma_L^*$$

- But we also know

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

- So: Γ_S depends on Γ_L (and visa-versa)
  – Must solve for both Γ_S and Γ_L simultaneously (a single solution that satisfies all the constraints)
  – Book goes through the algebra in some detail…

Maximum Gain Design, Cont.

- Bottom line: there is a unique solution for Γ_S and Γ_L

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

Where:

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

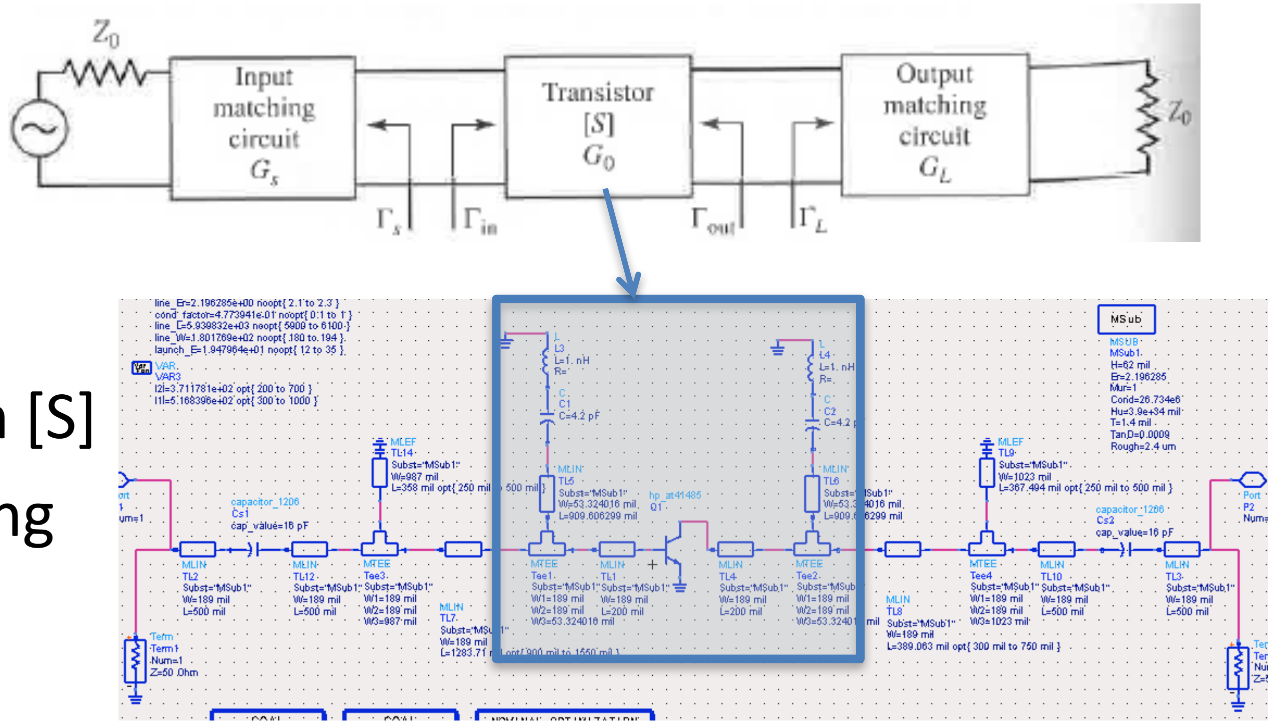
$$C_1 = S_{11} - \Delta S_{22}^*$$

$$C_2 = S_{22} - \Delta S_{11}^*$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21} \quad (\det(S))$$

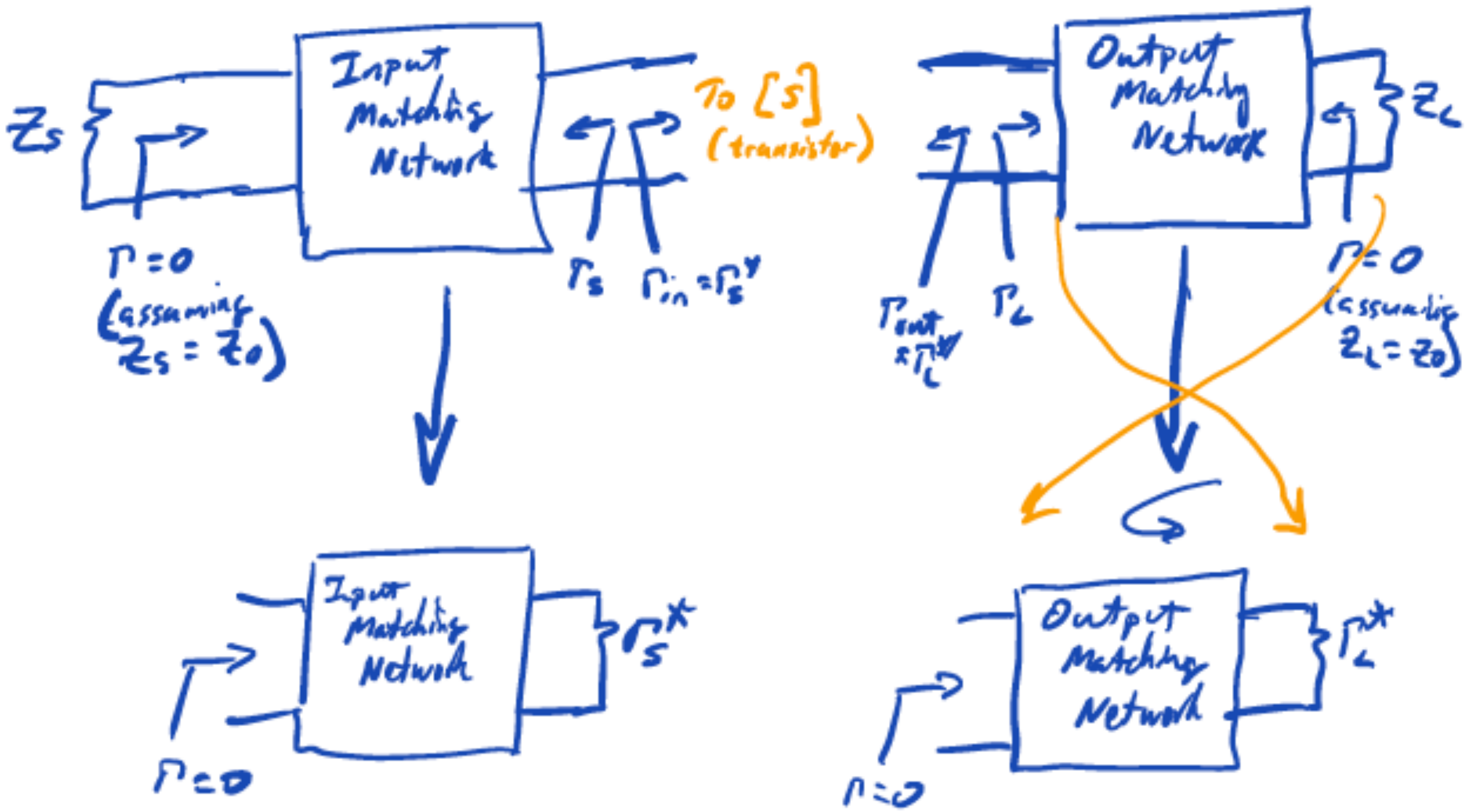
Maximum Gain Design, Cont.

- So: what to do with this?
- Find [S] of amplifying element (transistor, usually plus bias networks)
- Compute Γ_S, Γ_L from [S]
- Design input matching network to go from source to Γ_S
- Design output matching network to go from load to Γ_L



Input & Output Matching

- So: maximum gain amplifier design is really designing two matching networks:



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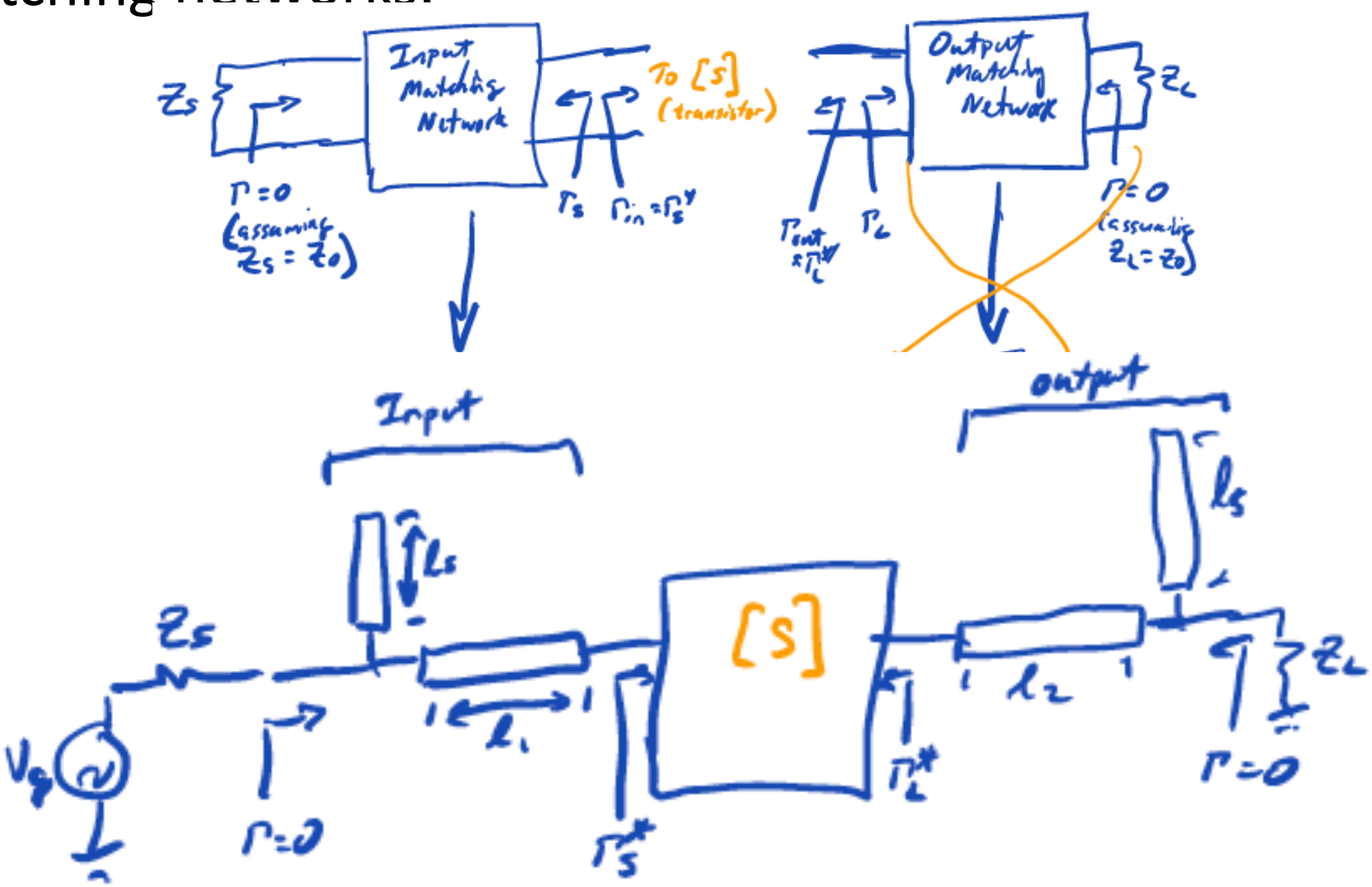


Specified Gain Design

- Key point: already know how to design an amplifier with maximum gain. Why would we ever go for less?
  - Need specific signal level
  - More likely: need more bandwidth than max. gain design can deliver; to get bandwidth, need to give up something else
- Approach: look at G_T expression:

$$G_T = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

- Note: 3 basic terms (source, S_{21}, and load)

$$G_T = G_S \cdot G_O \cdot G_L$$

Specified Gain Design, Cont.

- But a little complicated: G_S depends on load, G_L depends on source
- Yuck. So let's make a simplifying assumption—that [S] is unilateral (e.g. $S_{12} = 0$). Then $\Gamma_{in}=S_{11}$ and $\Gamma_{out}=S_{22}$
- Net result: $$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2};\ \ G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$
- So how can we set/change the gain? Introduce (intentionally) mismatch at source and/or load

Specified Gain Design, Cont.

- For maximum gain, conjugate match at both input and output: $\Gamma_S = S_{11}^*$, $\Gamma_L = S_{22}^*$ (remember, we're assuming unilateral)
- So the maximum G_S, G_L become:

$$G_{S,\max} = \frac{1}{1-|S_{11}|^2}; \quad G_{L,\max} = \frac{1}{1-|S_{22}|^2}$$

- It turns out (read: after much algebra, you can show if you are lucky and factor things just exactly right) that:

constant $|G_S|$ and $|G_L|$ plot as circles in the Γ_S and Γ_L planes

Specified Gain Design, Cont.

- These are the “constant gain circles”
- Mathematically: $g_s = \frac{G_S}{G_{S,\max}}; \ \ g_L = \frac{G_L}{G_{L,\max}}$

Source side:

$$C_S = \frac{g_s S_{11}^*}{1-(1-g_s)\left|S_{11}\right|^2}$$

$$R_S = \frac{\sqrt{1-g_s}\,(1-\left|S_{11}\right|^2)}{1-(1-g_s)\left|S_{11}\right|^2}$$

Load side:

$$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)\left|S_{22}\right|^2}$$

$$R_L = \frac{\sqrt{1-g_L}\,(1-\left|S_{22}\right|^2)}{1-(1-g_L)\left|S_{22}\right|^2}$$

C: coordinates (on imaginary plane) of center of circle
R: radius

Specified Gain Design, Cont.

- So: idea is to figure out how much mismatch to introduce at input, output to get desired gain
- Total gain: $G_T = G_S \cdot G_O \cdot G_L$
  - Designer chooses G_S, G_L so that G_T comes out at value desired (but remember, $G_S \leq G_{S,max}$; $G_L \leq G_{L,max}$)
  - In principle, any combination is ok (results in the correct gain)
  - But usually want to get something in exchange for the trade (e.g. bandwidth)
  - That additional trade is where design insight comes in

Example: Specified Gain

- Example 12.4 in Pozar is a good example
- Summary: design an amplifier at 4 GHz with 11 dB gain; try to maximize the bandwidth
  - Given: transistor S-parameters: $S = \begin{bmatrix} 0.75\angle -120 & 0 \\ 2.5\angle 80 & 0.6\angle -70 \end{bmatrix}$
- Can compute $G_{S,max}$; $G_{L,max}$

$$G_{S,\max} = \frac{1}{1-|S_{11}|^2}; \quad G_{L,\max} = \frac{1}{1-|S_{22}|^2}$$

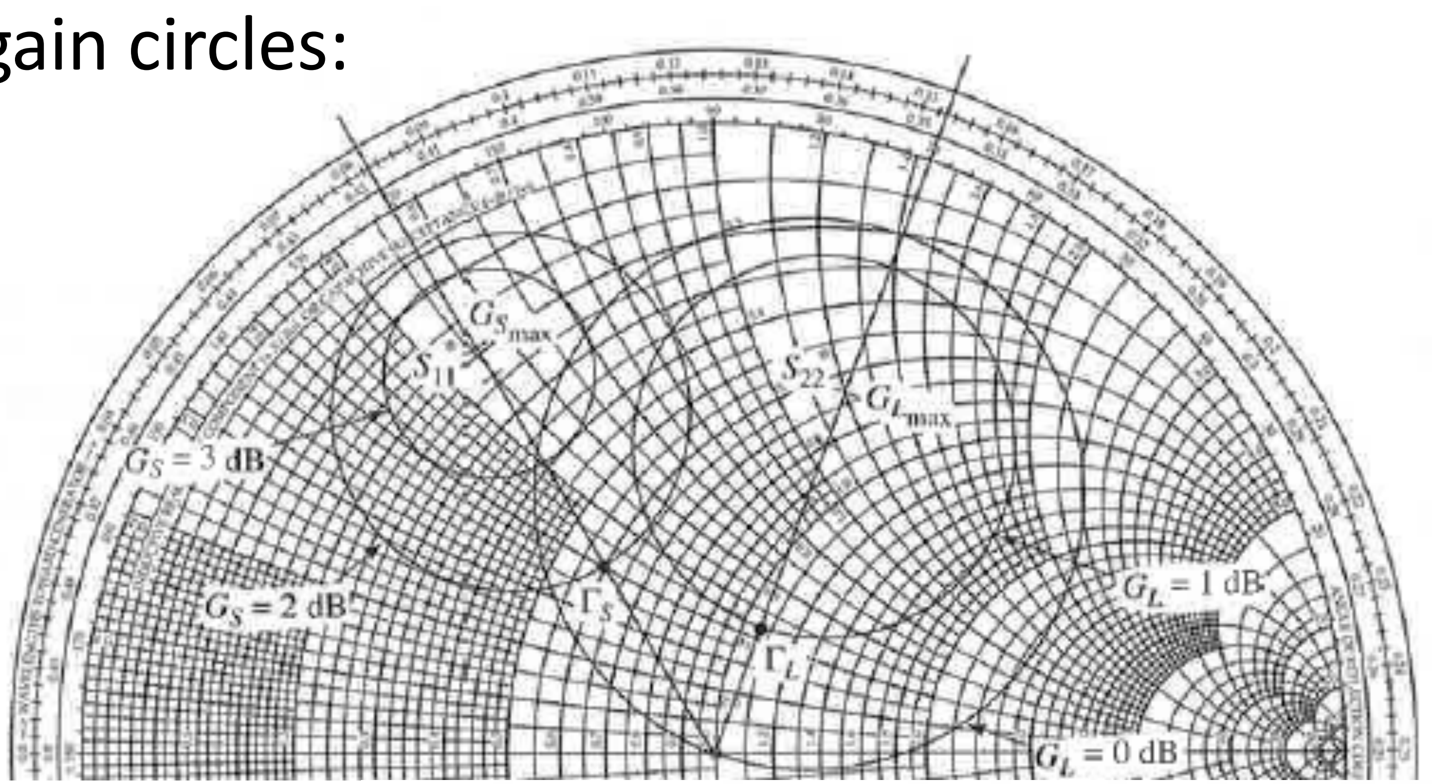
Numerically: $G_{S,max}$ = 2.29 = 3.6 dB; $G_{L,max}$ = 1.56 = 1.9 dB

Go = $|S_{21}|^2$ = 6.25 = 8 dB

- Max gain: 13.5 dB. We can give up 2.5 dB in combination of G_S, G_L

Example: Specified Gain (cont.)

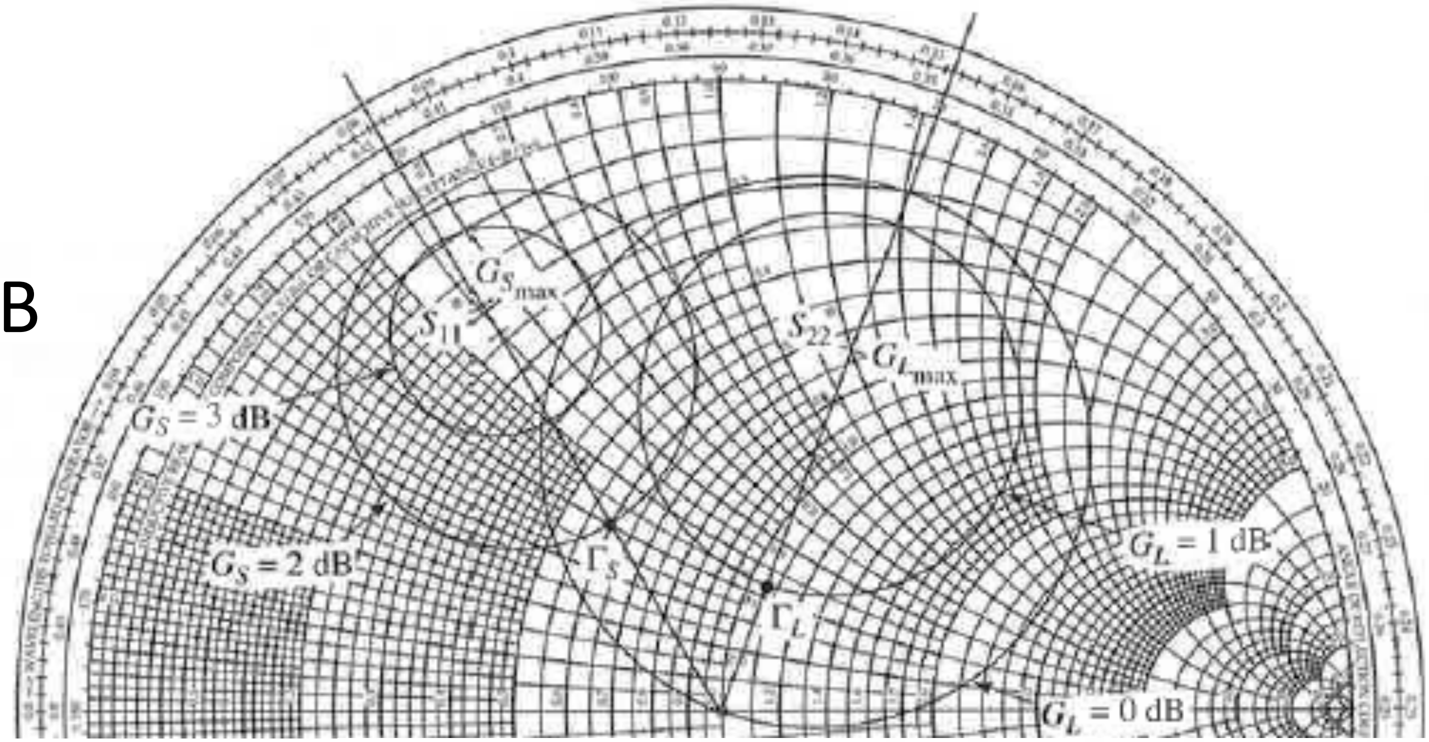
- Pick a few choices for G_S, G_L
- Plot the gain circles:



$G_S = 3$ dB	$g_S = 0.875$	$C_S = 0.706\angle 120°$	$R_S = 0.166$
$G_S = 2$ dB	$g_S = 0.691$	$C_S = 0.627\angle 120°$	$R_S = 0.294$
$G_L = 1$ dB	$g_L = 0.806$	$C_L = 0.520\angle 70°$	$R_L = 0.303$
$G_L = 0$ dB	$g_L = 0.640$	$C_L = 0.440\angle 70°$	$R_L = 0.440$

Example: Specified Gain (cont.)

- OK…
- Remember:
  $G_{S,max}$ = 3.6 dB
  $G_{L,max}$=1.9 dB
- Options:
  - G_S=3 dB,
    G_L=0 dB
  - G_S=2 dB
    G_L=1 dB
  - Both of these sets are 2.5 dB less than the maximum, give us 11 dB total



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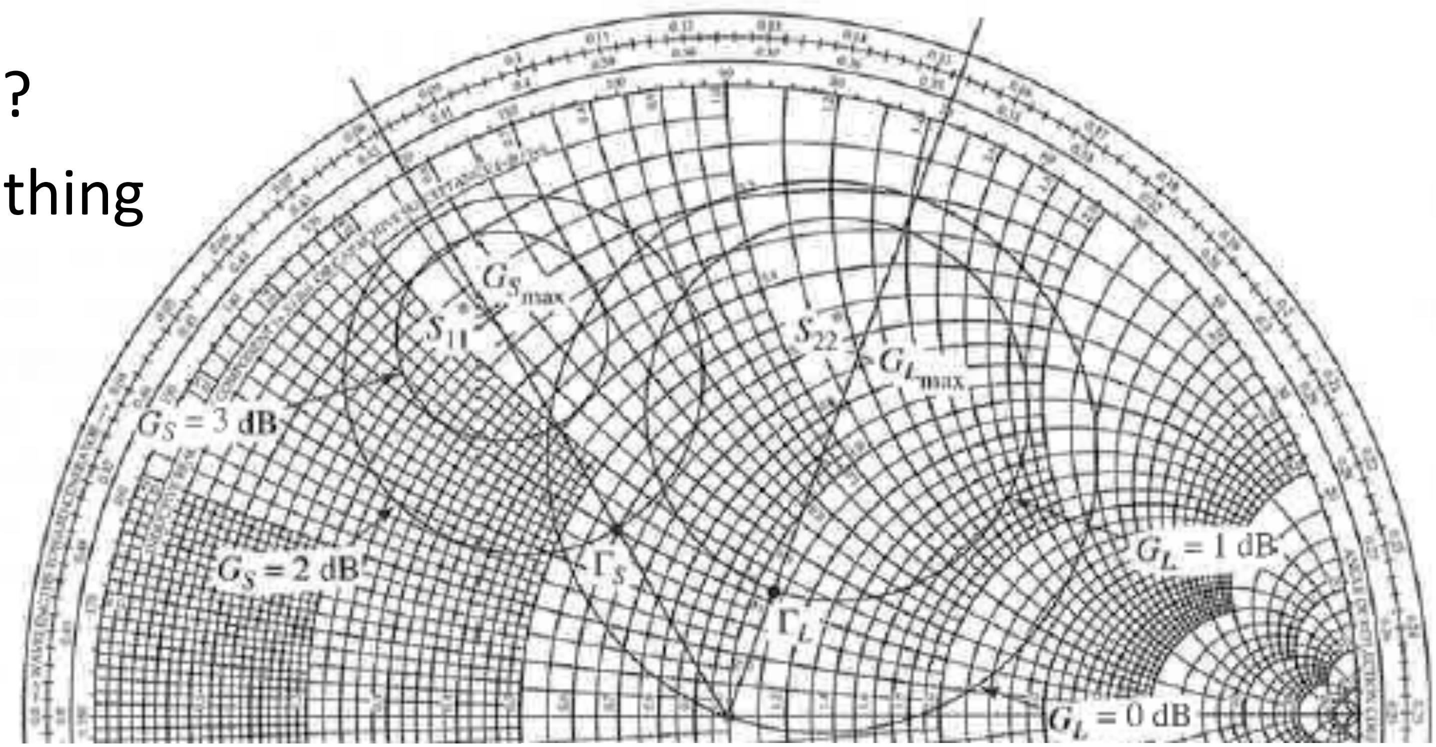
Example: Specified Gain (cont.)

- How to choose?
- Trade for something else—like bandwidth
- Recall: for largest bandwidth, matching network should not "work hard"



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- Pick Γ_S, Γ_L nearest center of Smith Chart—"laziest" matching network
- If pick roughly equal input, output matching networks, overall BW maximized (filters in cascade)

Low Noise Amplifier Design

- Last case: low noise amplifier
- Noise limits many systems (radio, wireline, fiber receivers, etc.)
- Performance dominated (usually) by first amplifier in cascade
- Unfortunate fact: best noise performance not obtained at maximum gain. Have to match for either noise *or* gain; a tradeoff is inevitable

Noise in Two-Port Networks

- Noise in amplifiers (as represented by 2-port networks) can be quantified by noise factor:

$$F = \frac{SNR_{in}}{SNR_{out}}$$

  – Noise factor is a measure of how much worse the SNR (power) is at the output compared to the input, due to noise added by the network

  – Noise factor depends on input termination (Γ_S):

$$F = F_{\min} + \frac{4R_N}{Z_O} \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{(1-\left|\Gamma_S\right|^2)\left|1+\Gamma_{opt}\right|^2}$$

Noise in Two-Port Networks

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$$F = F_{\min} + \frac{4R_N}{Z_O} \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{(1 - \left|\Gamma_S\right|^2)\left|1 + \Gamma_{opt}\right|^2}$$

  – Γ_{opt}, R_N, F_{min} are "noise parameters" of the device

- Can show (if you are patient) that constant F contours are (gasp) circles in the Γ_S plane:
- C_F is center (complex plane), R_F is radius

$$C_F = \frac{\Gamma_{opt}}{N+1}$$

$$R_F = \frac{\sqrt{N(N + 1 - \left|\Gamma_{opt}\right|^2}}{N+1}$$

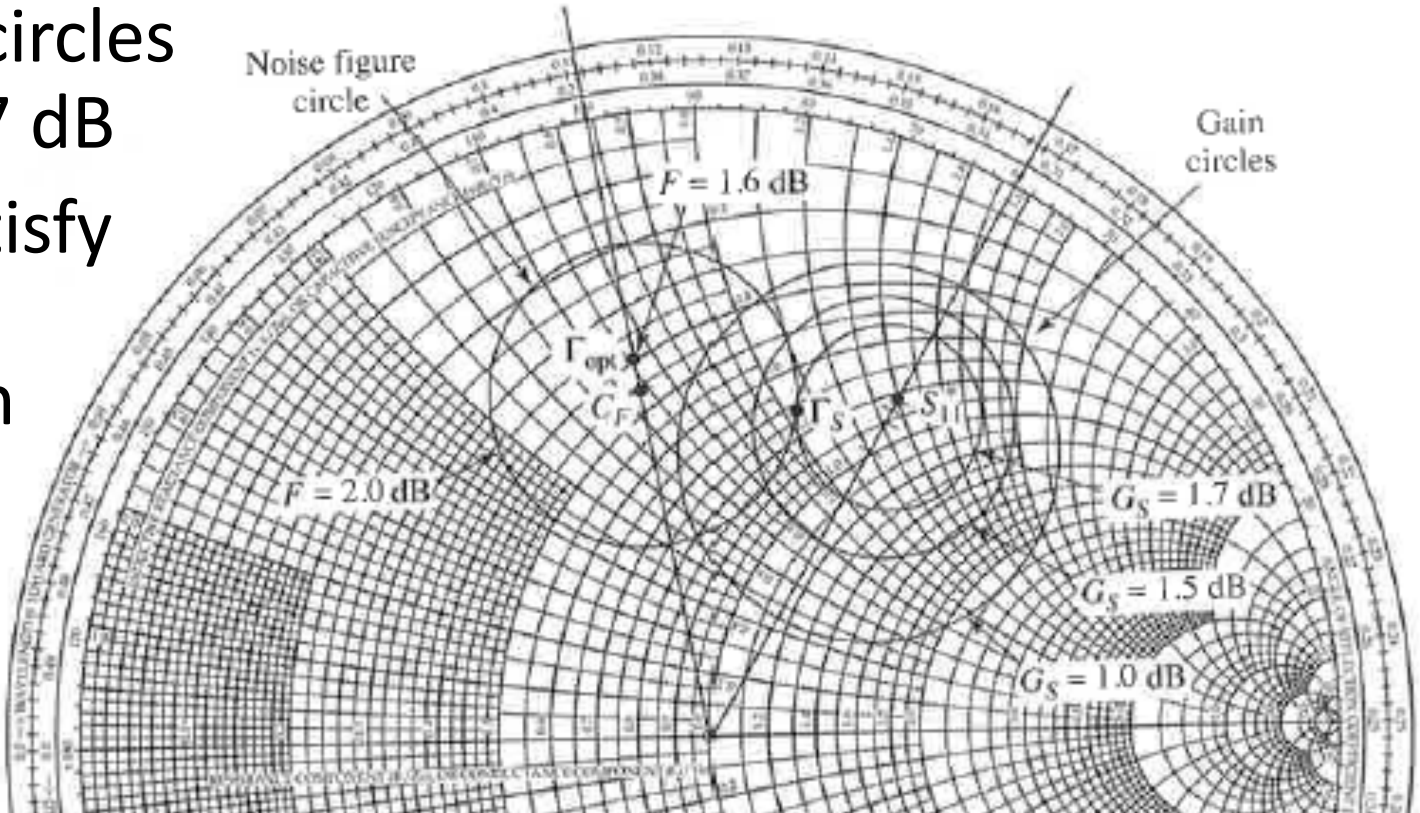
$$N = \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{1 - \left|\Gamma_S\right|^2} = \frac{F - F_{\min}}{4R_N / Z_O}\left|1 + \Gamma_{opt}\right|^2$$

Low Noise Amplifier Design

- Example 12.5 in Pozar:
  - Design an amplifier with 2 dB noise figure and maximum gain (consistent with this noise level)
  - Plot Noise figure circles
  - Note that noise figure is related to Γ_S; but gain is too (specified gain: G_S term)
  - So we look at both constant gain and noise figure circles on the same plot:

Low Noise Amplifier Design (cont.)

- Noise figure: NF=2 dB circle
- Several G_S circles 1.0, 1.5, 1.7 dB
- Need to satisfy both: find intersection between NF, highest gain G_S circle



- What about output? Conjugately match once input is finalized