Today’s Topic: More Lumped-Element Circuit Models

- Recall:
  - We discussed a wire (inductor), resistor (series L, parallel RC) last time
- Plan: round out our “library” of components
  - Capacitor, inductor
  - Examine the impact of parasitic elements on circuit performance
- Move on to distributed circuits

Recap: Circuit Models for Components

- Start with workhorse passives: R, L, C
- Low frequency regime ($\ell<\lambda/100$):
  - Easy: just like EE 20242: $V=I*R$, $V=j\omega L*I$, $I=j\omega C*V$
  - Nothing new
- "lumped element" models ($\lambda/100<\ell<\lambda/10$)
  - Phase/delay is important, need to augment our treatment to capture that, but would like it to be simple
  - We'll work up models for components
- Even a wire isn't so simple—not an ideal short
  - Ideal short: phase delay = 0; wire of length $\lambda/10$, phase delay ~36°
  - Fix: model as inductance. Empirical formula (very handy…)

$$L(\mu H) = (0.002\ell)\ln(4h/d)$$

  - ℓ=length (in cm), d=diameter, h= height above ground plane

Short Wire

$$L(\mu H) = (0.002\ell)\ln(4h/d)$$

- ℓ=length (in cm), d=diameter, h= height above ground plane

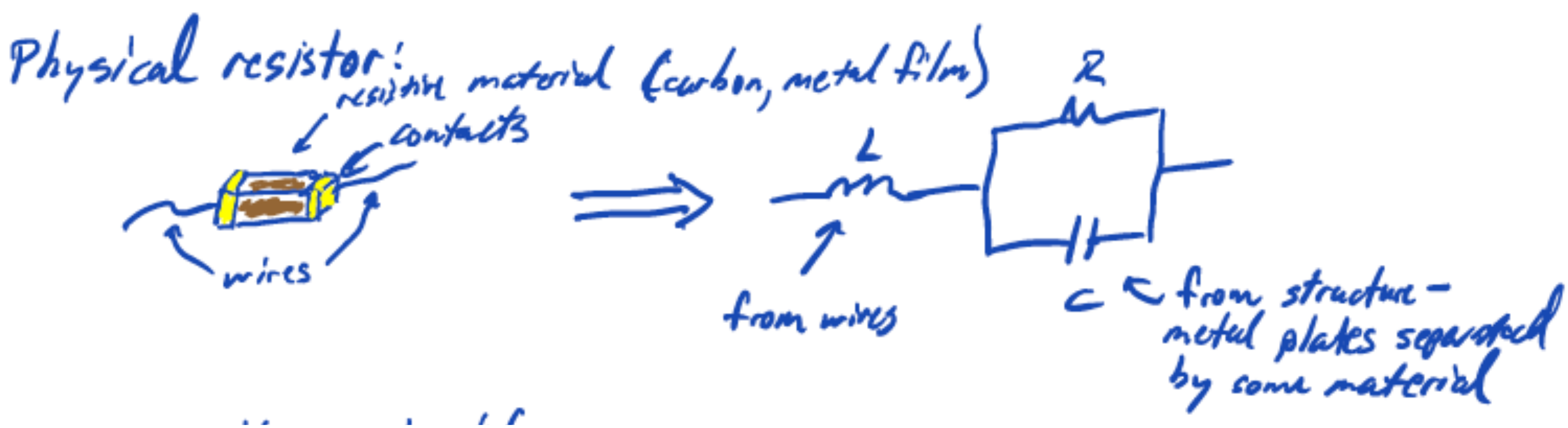
- A numerical example:
  - #22 wire (like for a breadboard): d=25.3 mils = 0.0643 cm

  (aside: microwave people use “mils” a lot; 1 mil=0.001”. Yes, inches)
  - h/d in range from 10/100 (inside ln, so not so sensitive)
  - ➢ L=7.4 nH/cm to 12 nH/cm

- Does this matter? nH seems small…
- Put this in a circuit context. Assume h/d=100 (12 nH/cm)
  - At 10 MHz: impedance of wire is $j\omega L$ = ~j 1 Ω/cm
  - At 100 MHz: impedance of wire is ~j 10 Ω/cm
  - Depending on what the rest of the circuit looks like, this could be nothing, or it could be a big deal (is it in series with 25Ω? Or 1000Ω?
  - Note it can start to matter at quite low frequencies (below 100 MHz)

Other Components: R, L, C

- But first some vocabulary:
  - Impedance, admittance, reactance, susceptance—be sure we're all on the same page
- Z (impedance) = R (resistance) + j X (reactance)
- Y (admittance) = G (conductance) + j B (susceptance)
- Y=1/Z
- Careful:
  - $G \neq 1/R$, $B \neq 1/X$!
  - Probably obvious if you think it through, but so tempting…
- Resistor: lumped element circuit model



Lumped Element R model

- This model is pretty general, for $\ell<\lambda/10$, but is surprisingly complex in response

Can sometimes simplify:

If R small ($\ll 100\,\Omega$), $R \| C$ dominated by R

$\rightarrow$ 

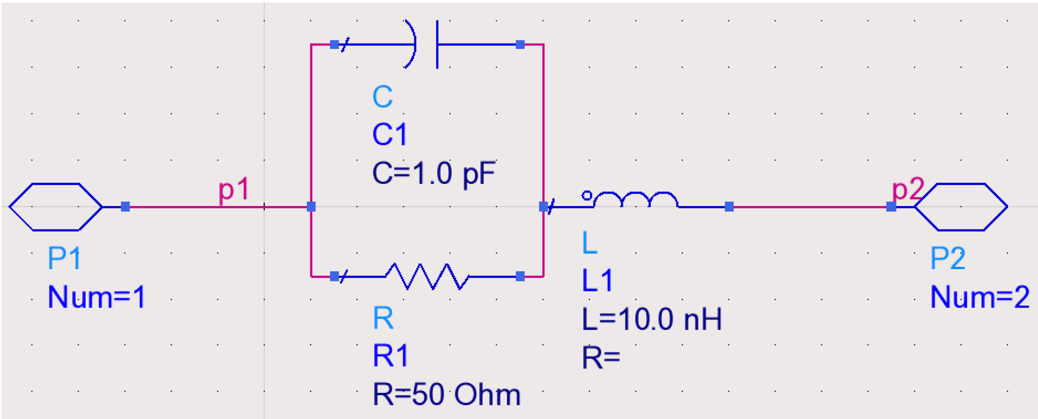
If R large ($\gg 100\,\Omega$) L not so important

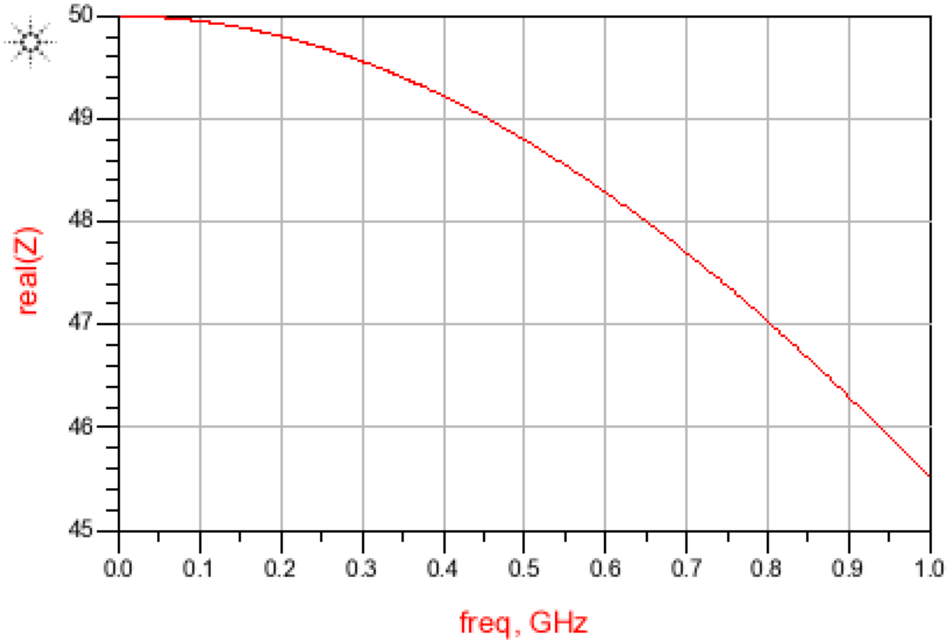
$\rightarrow$ 

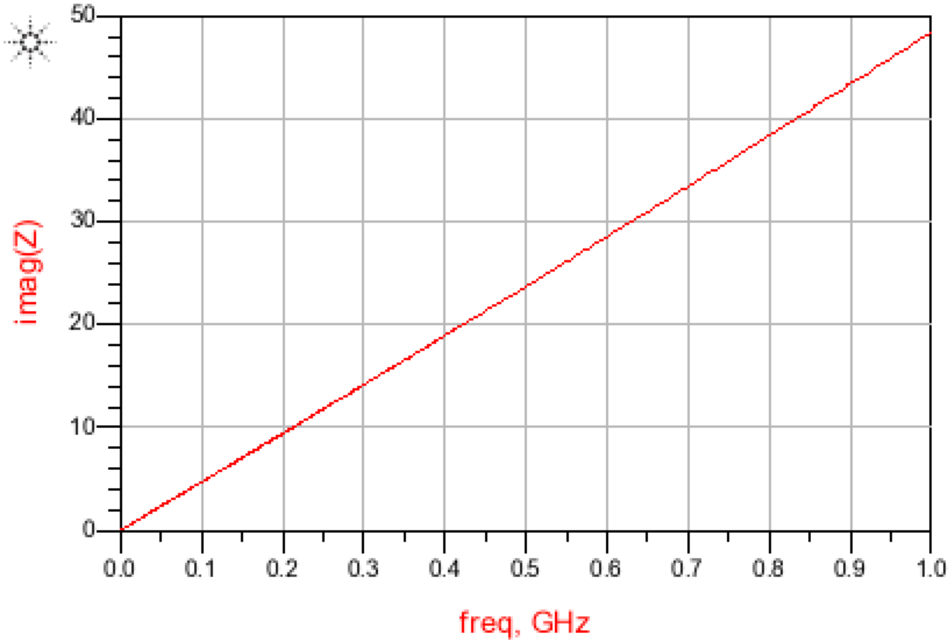
But these are rough bounds. For intermediate values or if in doubt, use full model.

Lumped Element R Examples

- Small-ish resistor:
  - 50 Ω
  - C = 1 pF
  - L = 10 nH
    ~5 mm of wire on each end)



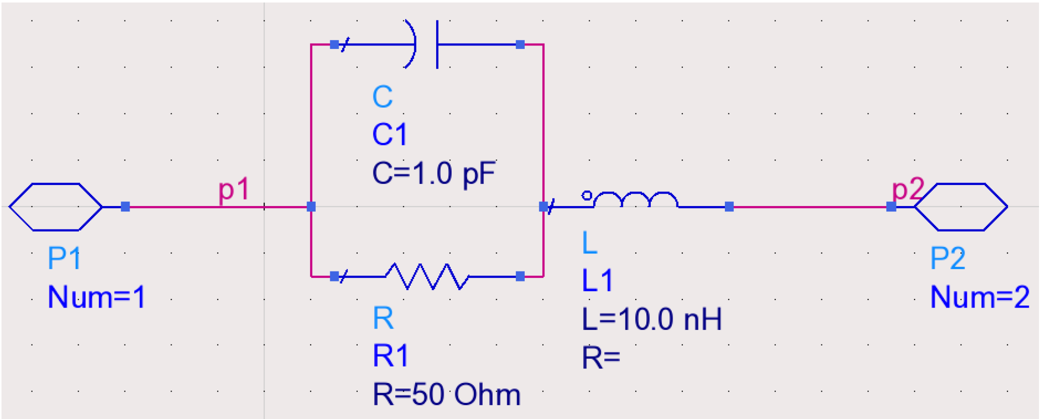


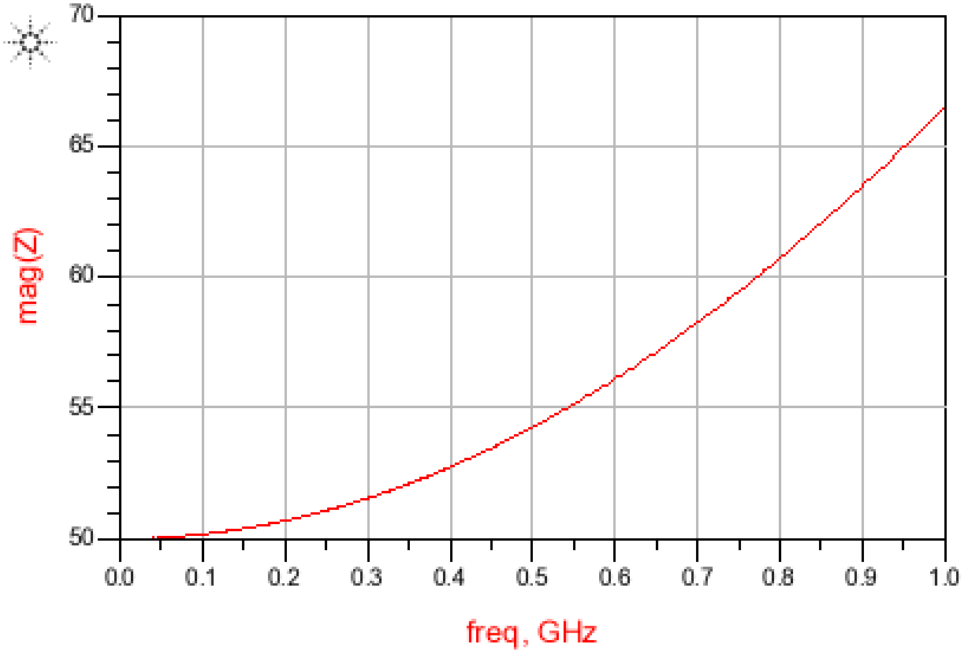


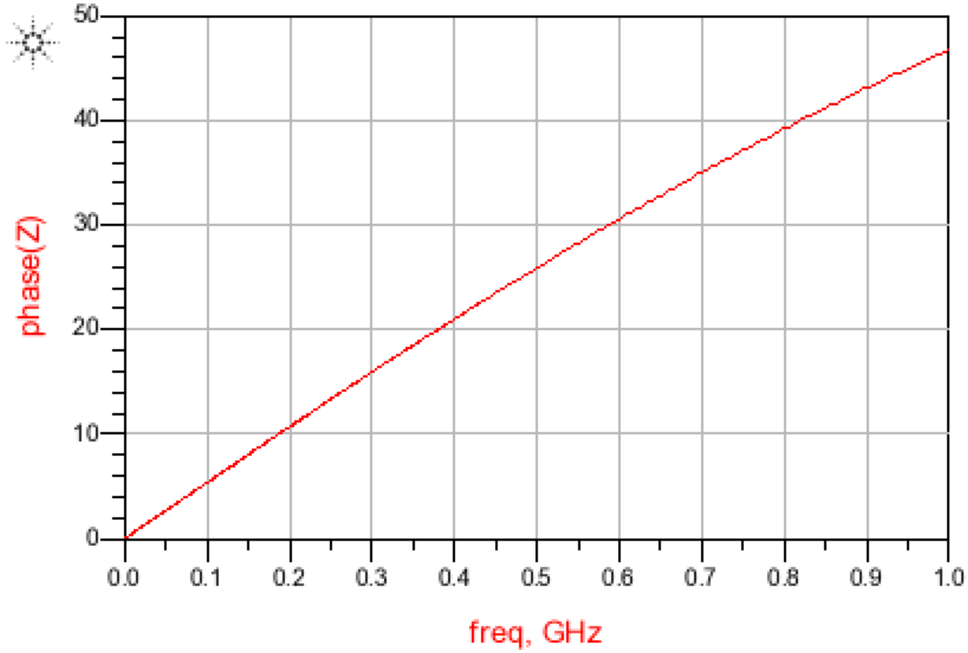
  - Real part not changed much, but significant imaginary part

Small Resistor—another look

- Same resistor, same data—but |Z| and angle
- Overall magnitude strongly affected; significant phase



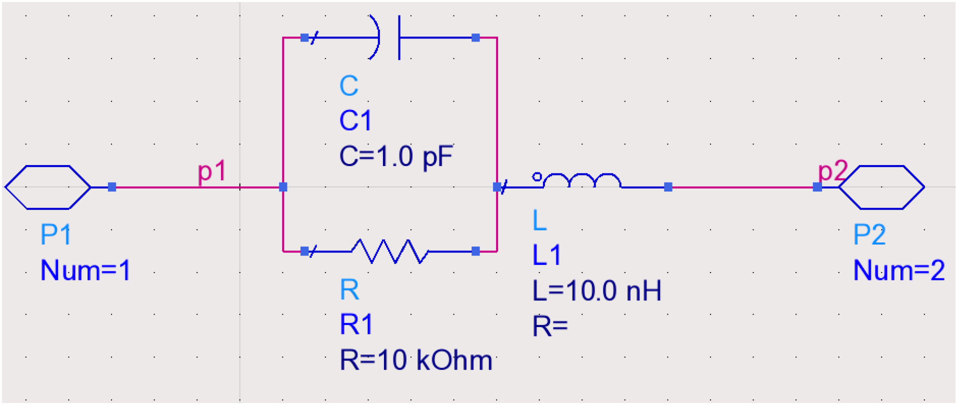


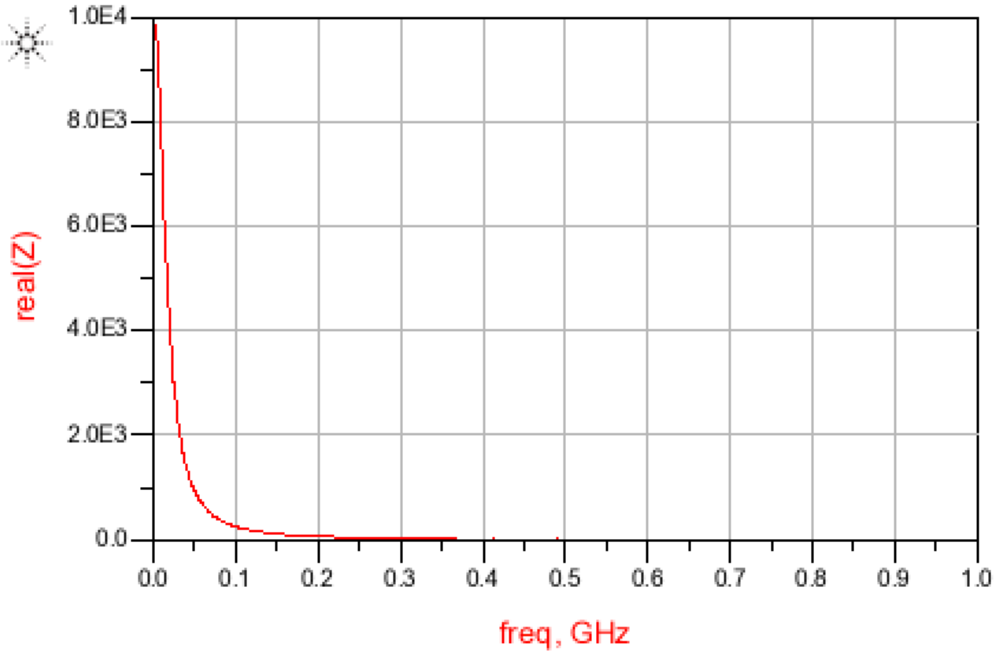


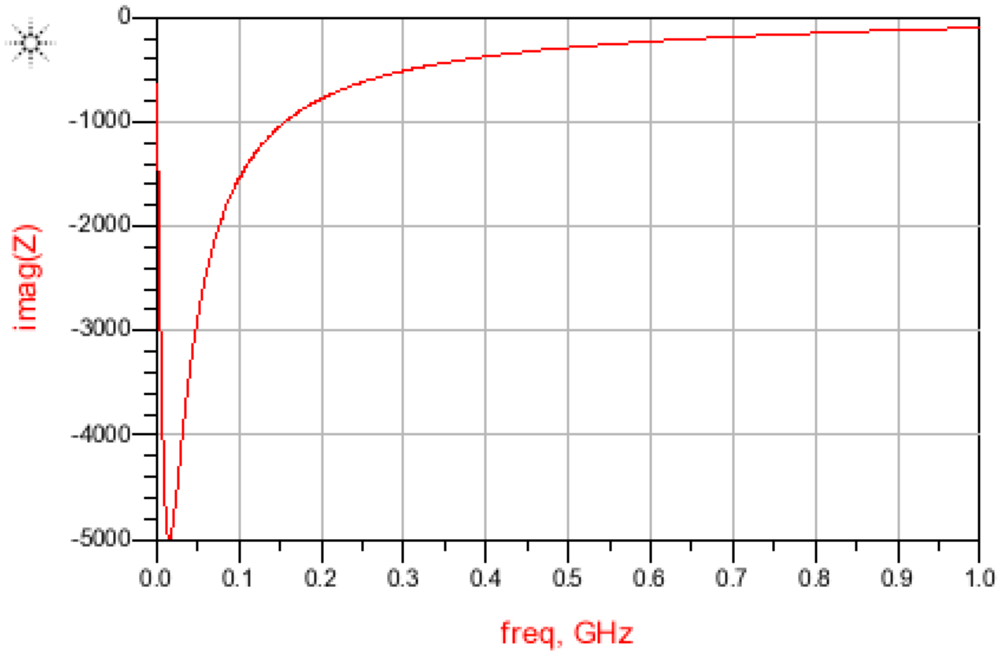
– What you see depends on what you look for

Large Resistor

- Large resistor:
  – 10 kΩ
  – 1 pF
  – 10 nH



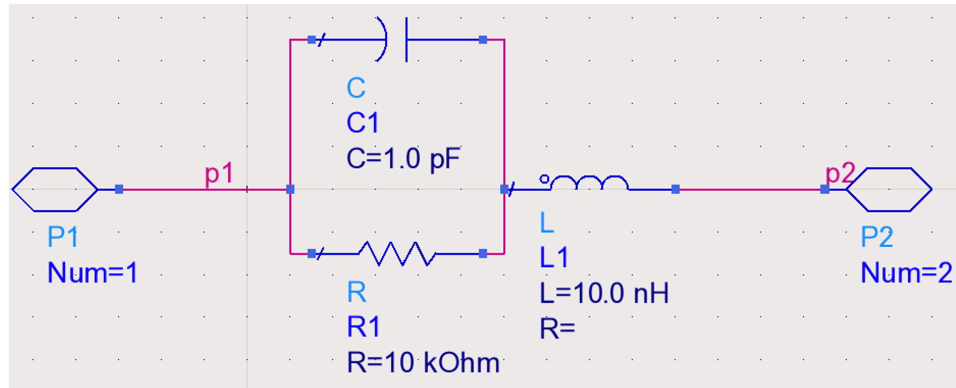


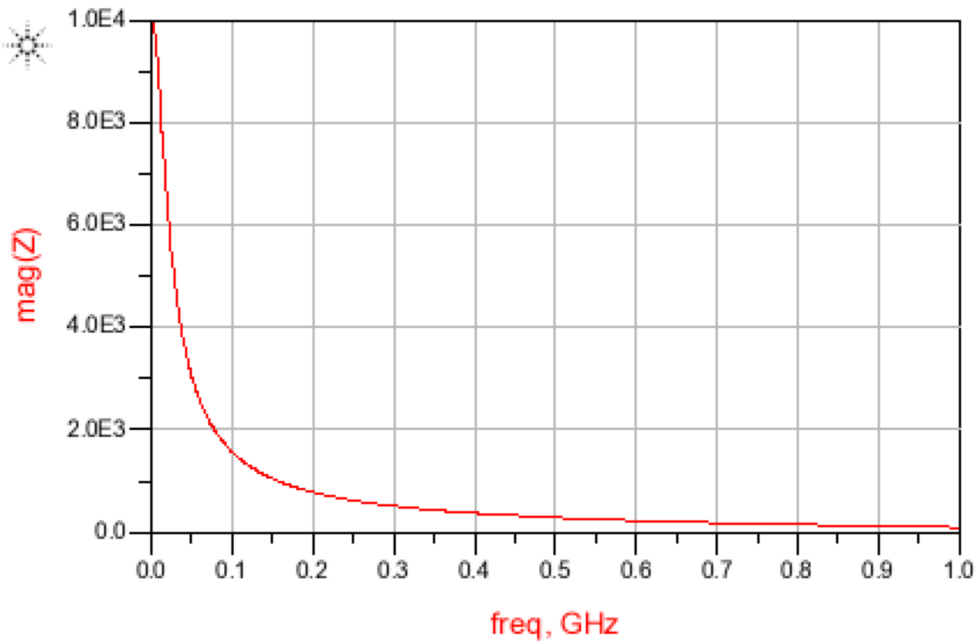


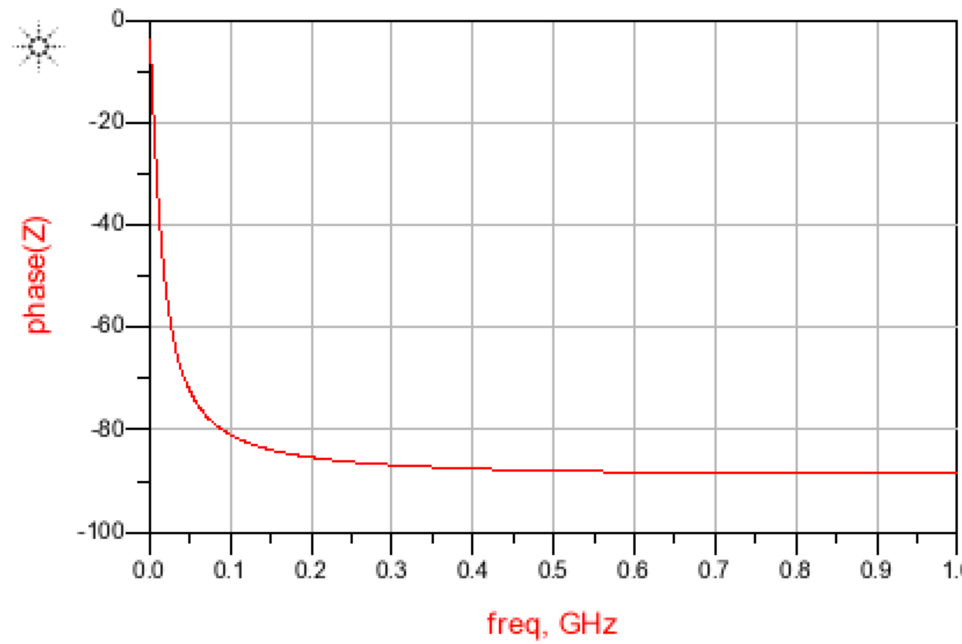
  – Real part falls off a cliff, imaginary part has big negative peak at very low frequencies; big resistors don't work well at RF…

Large Resistor – another look

- Mag/angle view often easier to interpret
- |Z| falling from shunt C
- Phase –> 90 °--capacitor



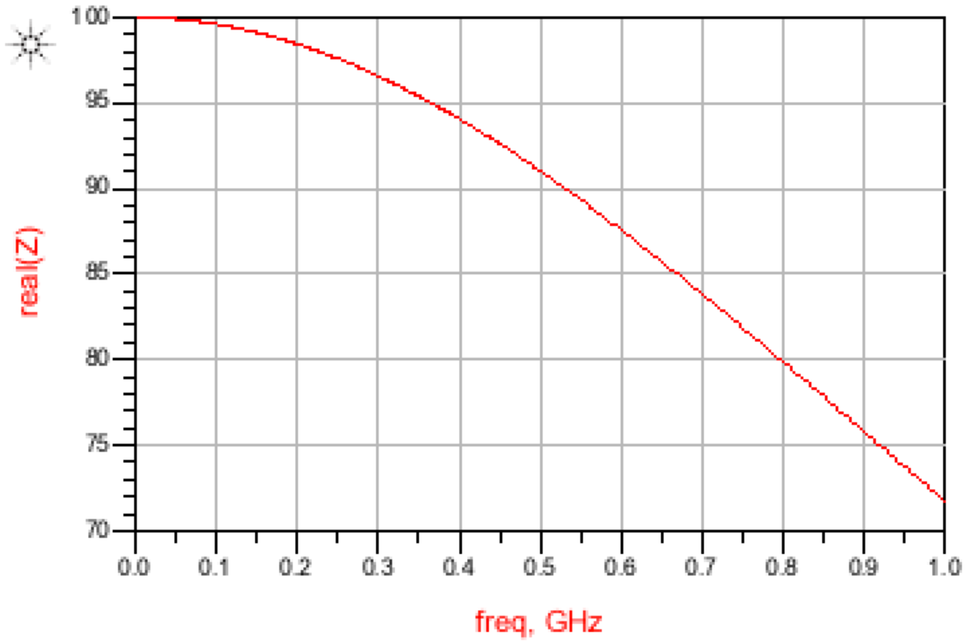




– Conclusion: big resistors don't work well at RF…

“Intermediate” R Example

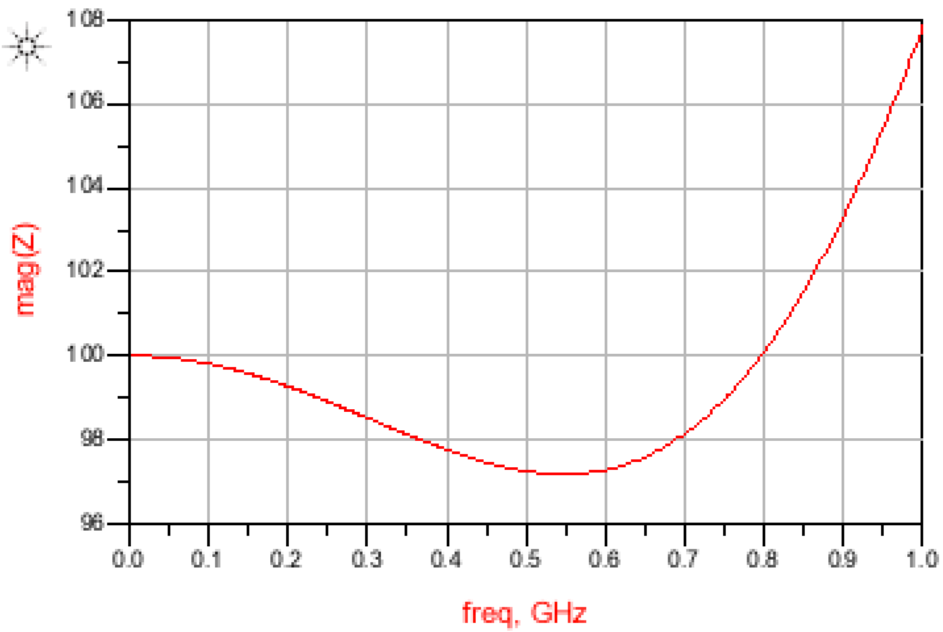
- Intermediate resistor: R = 100 Ω, C=1 pF, L=10 nH



- Real part falls more dramatically than small R, less so than large
- Imaginary part comparable to real part at high frequencies

“Intermediate” R – another look

- Intermediate resistor: R = 100 Ω, C=1 pF, L=10 nH

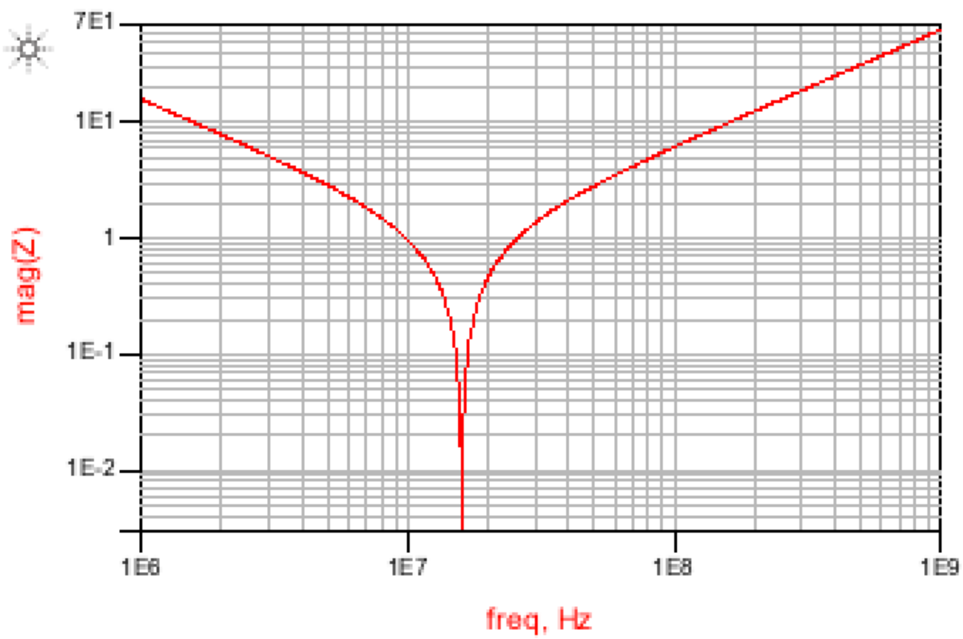


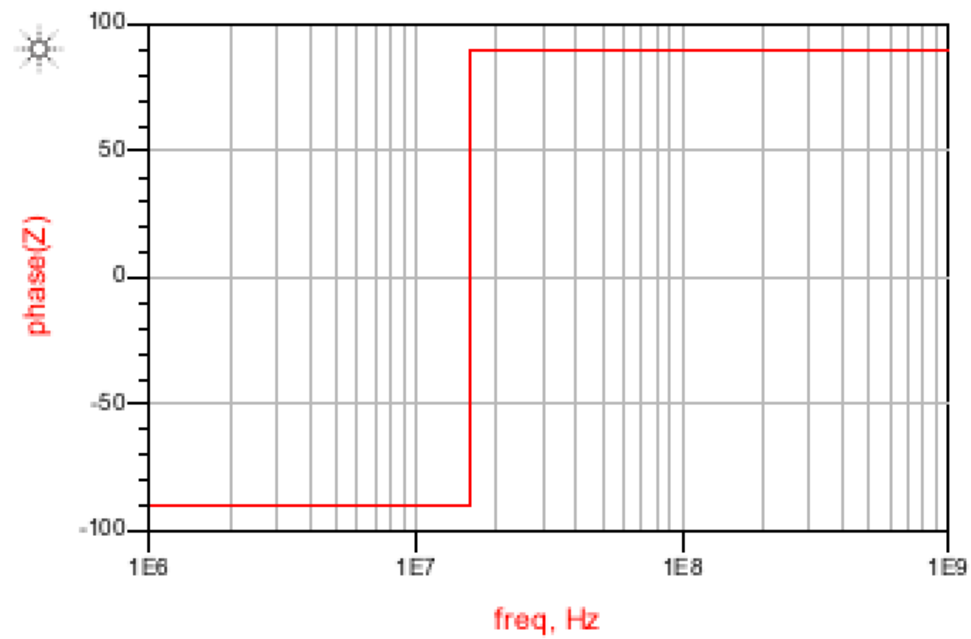
- Note: |Z| can be larger or smaller than DC resistance
- Not captured by either approximation—need full model
- Life is not so simple at RF…

Capacitors

- Real-world capacitors aren't ideal either…

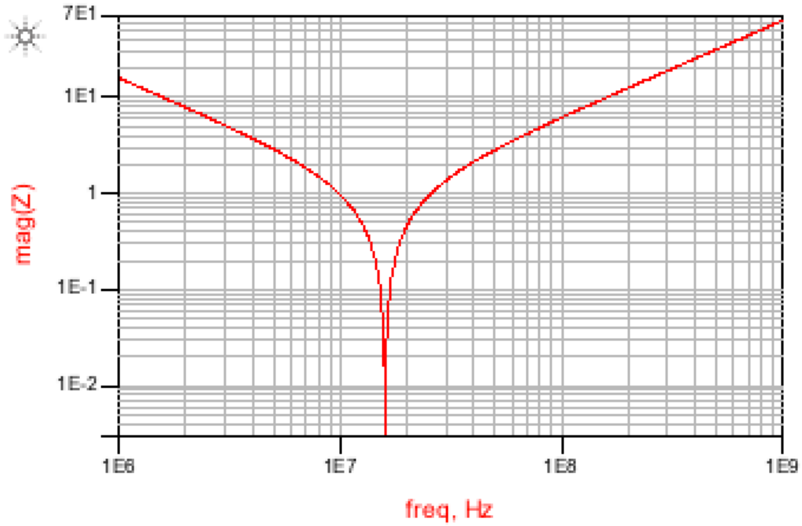
- Performance: C=0.01 μF, L=20 nH (1 cm of wire at each end)

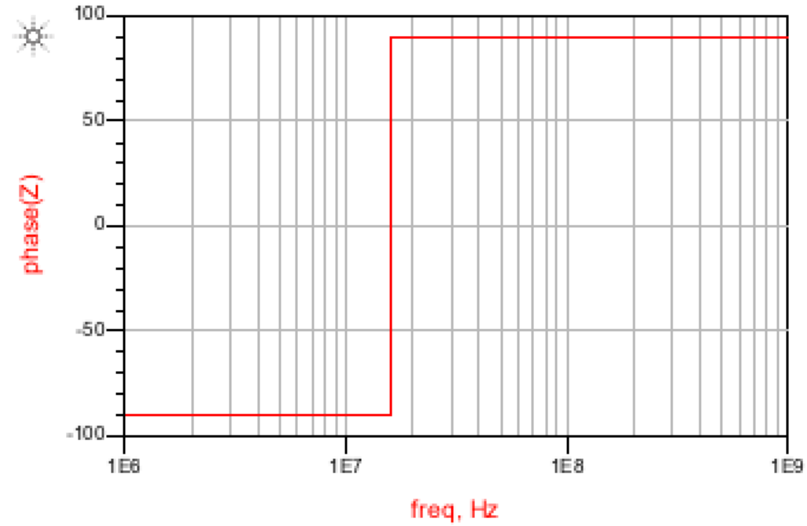




Capacitors

- Note big dip (heads to zero—huge hole on log plot) and abrupt flip in phase





f_s = "series resonance" $\rightarrow Z = 0$

$$Z = \frac{1}{j\omega C} + j\omega L = j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z = 0 \Rightarrow \omega_s = \frac{1}{\sqrt{LC}} \rightarrow f_s = \frac{1}{2\pi\sqrt{LC}}$$

- Below f_s – reactance < 0 (like C); above f_s, reactance > 0 (L!)
- Ideal C: X=-1/(ωC) – straight-line part below ~4 MHz or so
- This behavior can be a problem or a help—but you have to know it is there!

Inductors

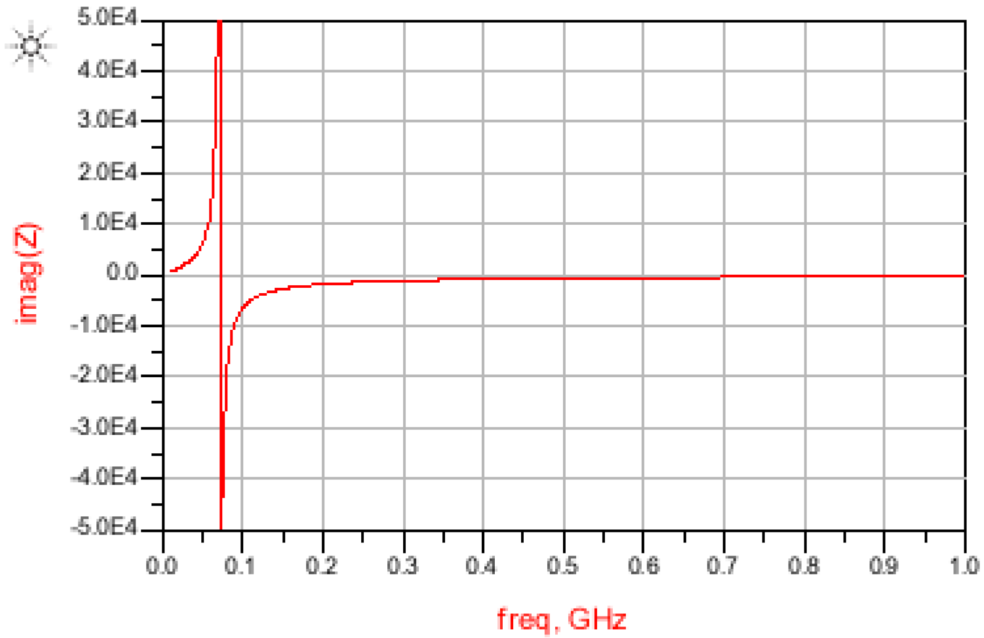
- In practice, inductors are often the least ideal of common passives.

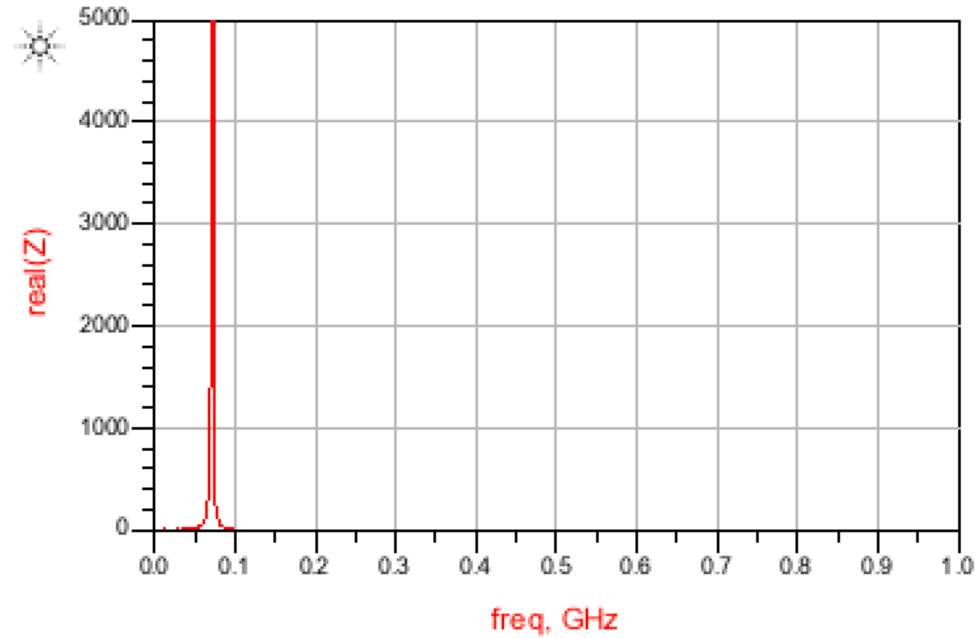
Physical inductor:

R — wire resistance (often use small wires)

C ← lumping all the turn-to-turn capacitance into one place

- Performance: L = 10 µH, C=0.5 pF, R = 5 Ω

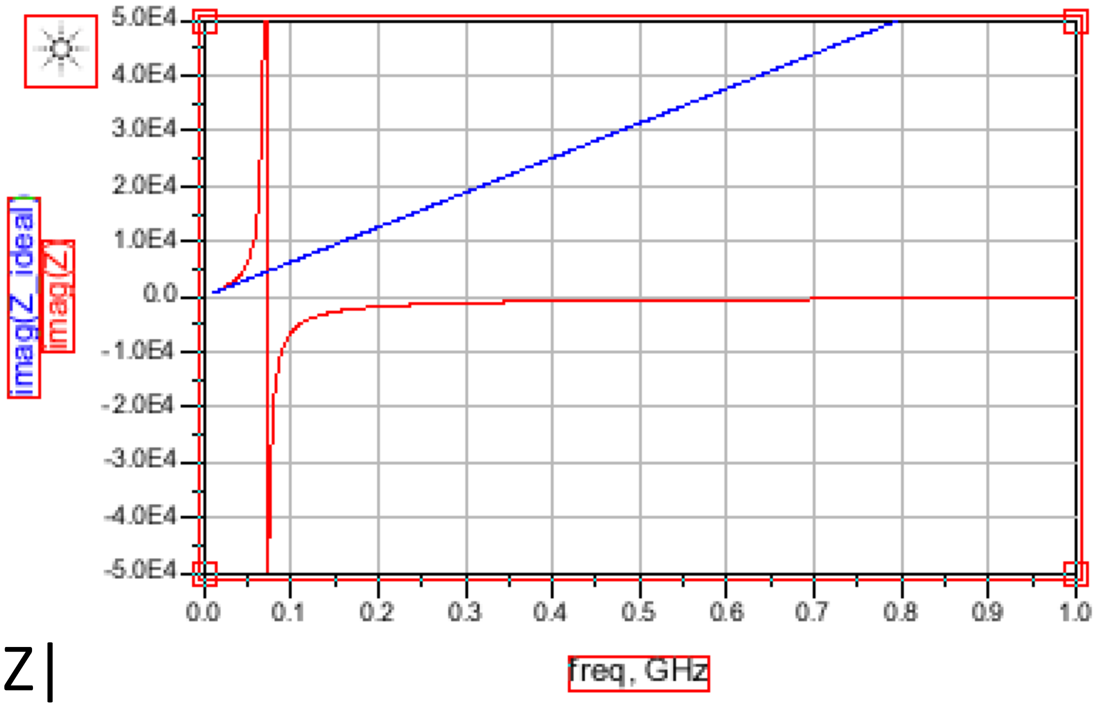




- OK…X > 0 at very low frequencies (X=ωL), but not very ideal

Inductors

- Let's compare: model vs. ideal L
- Zoom in on low-freq. range
- Plot X for ideal L ($X=\omega L$, $L=10\ \mu H$) and full model together
- Matches only at very low frequencies
- Big peak (in real and imaginary part; also in |Z|

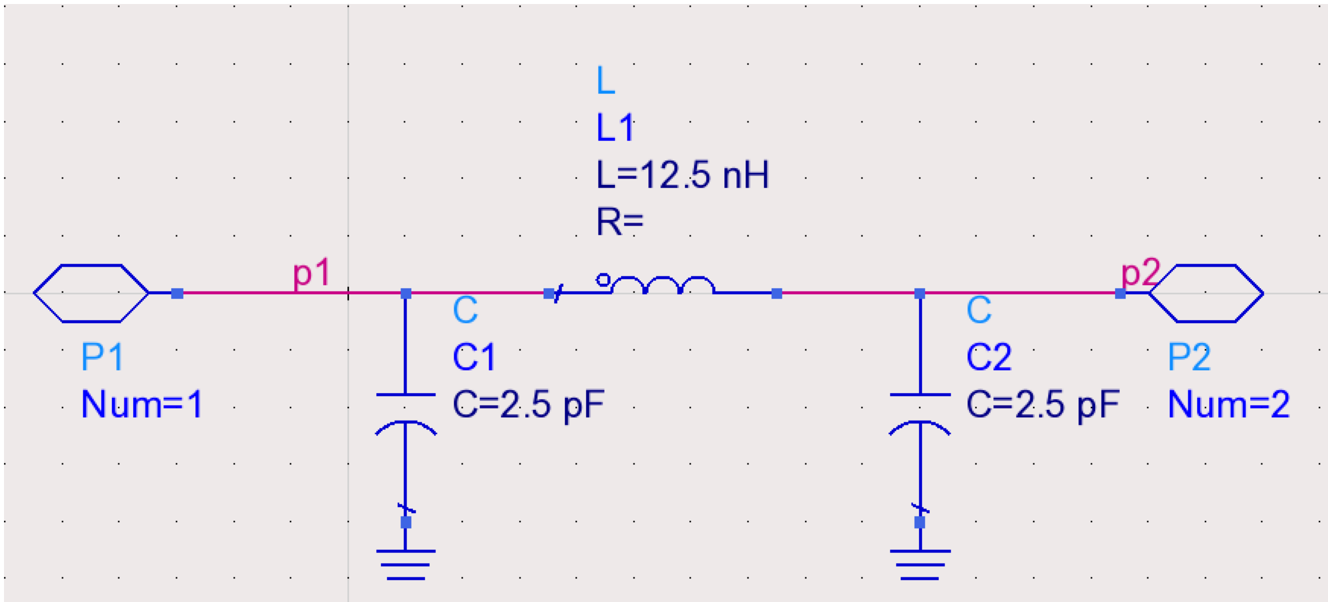


f_p = "parallel resonance" $= \frac{1}{2\pi\sqrt{LC}} \rightarrow |Z| \rightarrow \infty$ at f_p

- Same formula as f_s for capacitor, but very different behavior
- Behavior is lousy if you wanted $X=\omega L$
- Great if you want a DC short and RF "open"—called a "choke"; probably actually more useful…

Impact on Circuits?

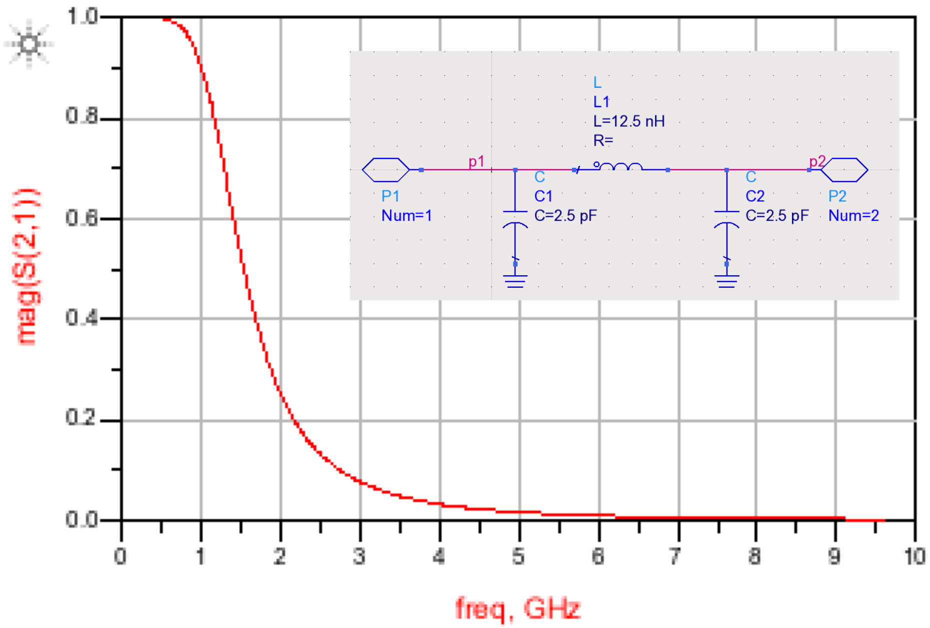
- So—does any of this matter much? After all, what we really care about is whether the circuit does what we want or not
- Example: RF low-pass filter



- Simple pi-network filter, easily designed using standard filter synthesis tools in CAD packages (ADS)
- Values computed automatically from filter specifications

Filter Performance

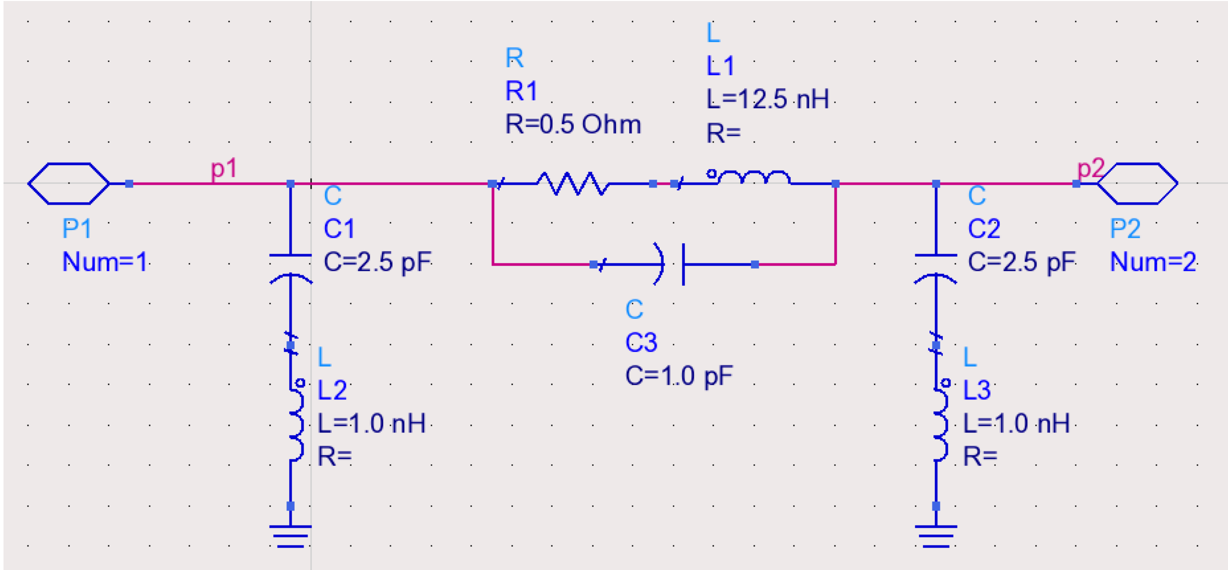
- Frequency response: RF low-pass filter

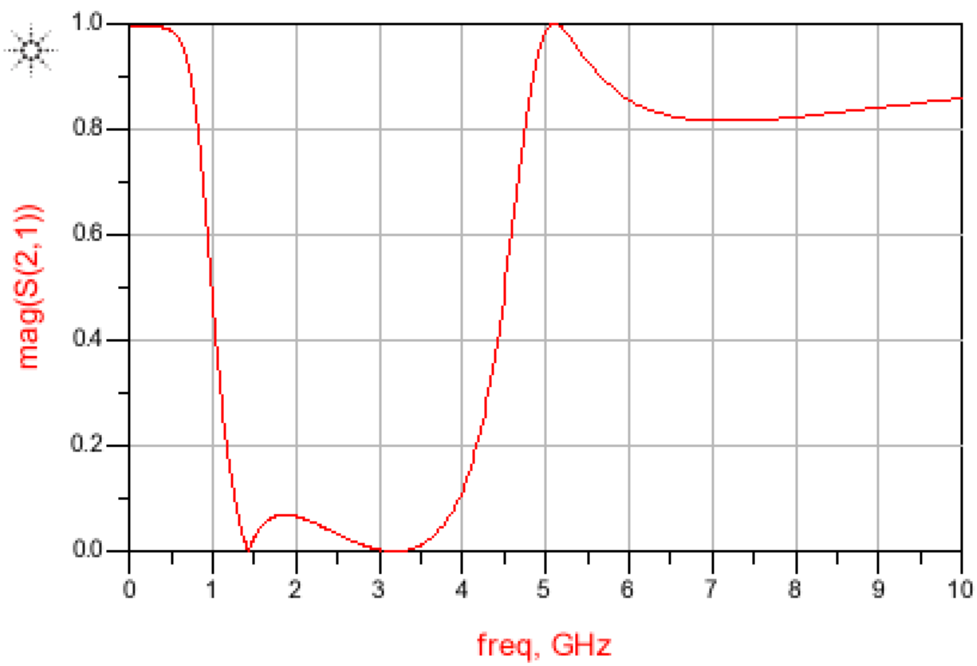


- Nice roll off, flat passband, what's not to like?

Real Filter Performance

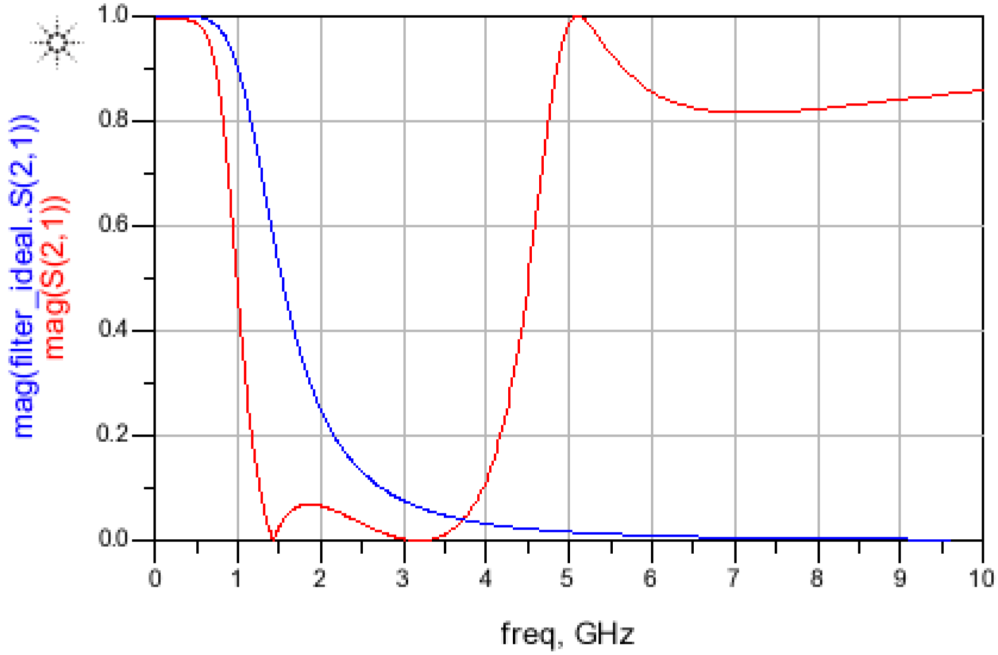
- Include the full model for each component
- Parasitics taken from typical surface-mount values
- Um…things are not so good

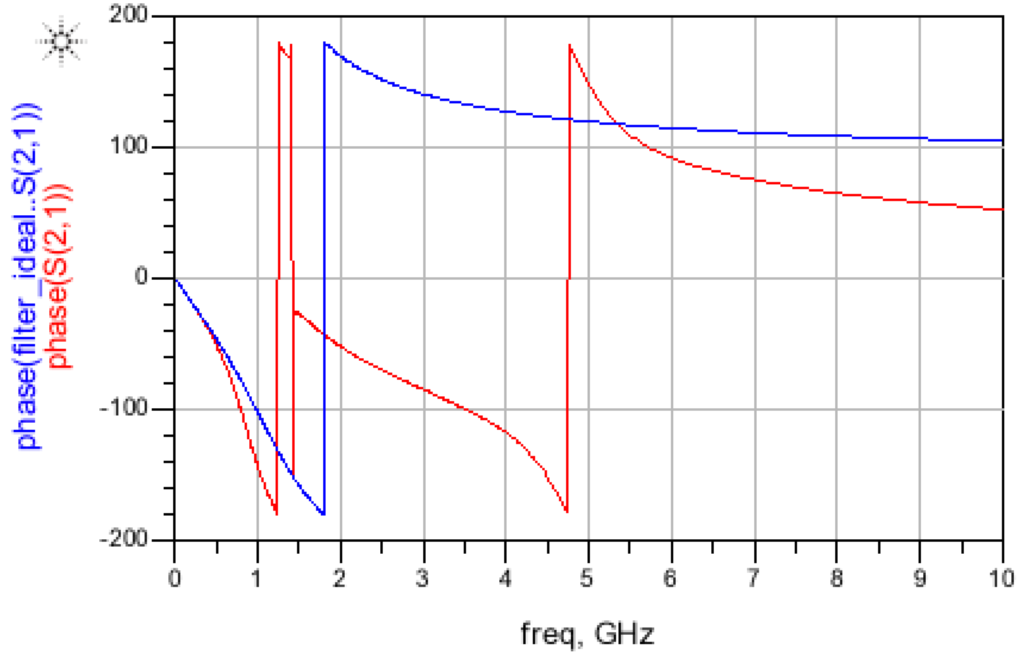




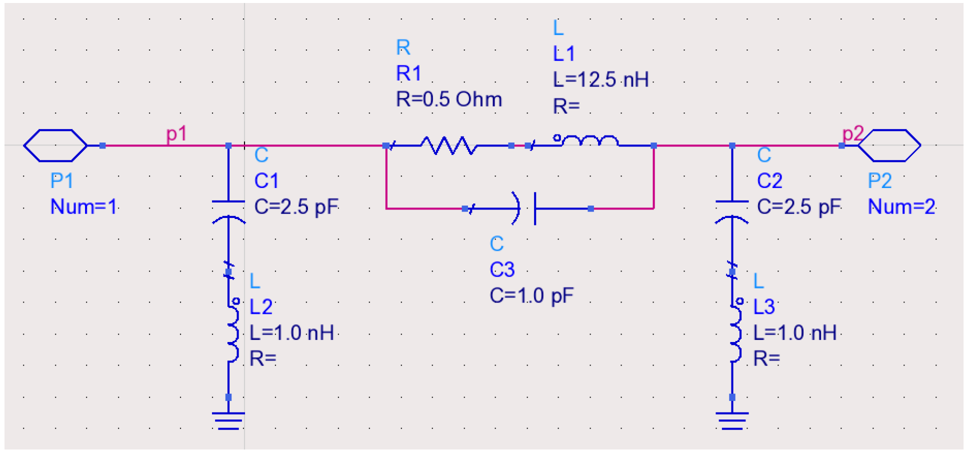
Real Filter Performance

- Comparison:





- Passband is narrower than before—if we wanted signals above 1 GHz to get through, um…
- “Second” passband at 5 GHz and above—if we wanted to block signals there, we blew it
- What’s wrong?



Recap: Lumped Element Models

- Have developed “lumped element” equivalent circuit models for typical R, L, C, plus wire
- Relies on $\ell<\lambda/10$, so not a property only of the component, but also of the signals
- Side note: be very cautious of vendor claims. They aren’t lying, but you need to understand what they mean…look at an example:
- http://www.usmicrowaves.com/res/ceramic/alumina_ceramic_al2o3_99ghz_thin_film_chip_resistor_re1020t10.shtml

Datasheet Details

- Here's the temping part: 99.47 GHz! That should be great for my mm-wave circuit at 94 GHz, right?
- Here's the real thing:

ELECTRICAL CHARACTERISTICS @25°C		
PARAMETER	VALUE	UNITS
Resistance range	0.1 to 1000000	Ω
Capacitance (maximum)	0.032	pF
Power dissipation	10.78	mW/°C
3dB frequency (R=50Ω)	99.47	GHz
Temperature Coefficient -55°C to 150°C	75	ppm/°C
Operating Temperature range	-55 to +150	°C
Maximum working voltage (R=50Ω)	0.73	V
Peak voltage at 25°C, 5 sec	1.03	V
Insulation Resistance @25°C	1e+12	Ω
RC constant (R=50Ω)	1.6	ps
Note: Power dissipation is provided for a temperature difference of 1°C between substrate and thermal (solder) joint.		

- 0.032 pF → $1/\omega C = 50\ \Omega$ at 99.47 GHz. Oh.
- So: at 99.47 GHz, $Z \neq 50\ \Omega$. $Z = 50\ \Omega \| -j50\ \Omega$. $|Z| = 35.4\ \Omega$, ang(Z)=-45°. Ooh. At 50 GHz? $|Z| = 44.7\ \Omega$, ang(Z)=-27°
- Caveat emptor? Of course…just do the analysis first, cut a purchase order second. They didn't hide anything…

Distributed Circuit Models

- So far: discussed "ideal" and "lumped element" models
  - "ideal": $\ell < \lambda/100$
  - "lumped element": $\lambda/100 < \ell < \lambda/10$
  - last one is "distributed" model: $\ell > \lambda/10$
  - Reminder: there's no fixed "frequency" cutoff—it is always size vs. frequency
- "Distributed": spatially-varying. So we're looking for a model that explicitly includes the geometry
  - Since the components are appreciable in size to wavelength, propagation effects not only important; may even dominate
  - Want our approach to be general, but as simple as possible: inherent trade-off between complexity and accuracy
  - Components: dimensions, materials & properties
  - Interconnects: dimensions, orientation, proximity to other elements, board and metal properties, …

Distributed Circuit Approaches

- The trade-off between accuracy & complexity leads to multiple approaches
  - Full field theory
  - Transmission line theory
- Full field theory:
  - Since the origin of the deviation from “regular old” circuit design is finite propagation of electromagnetic waves: use Maxwell’s equations to explicitly include propagation
  - Approach: use Maxwell’s equations, boundary conditions (geometries), material properties
  - Solve for E, H fields everywhere (two vector fields—6 complex components at each position, frequency)
  - Use E, H to find current, voltage vs. position (reduce 6 components to 2 complex scalars)
  - Difficult by hand; time consuming (by computer), requires real effort

Distributed Circuit Approaches

- Transmission line theory
  - Can be viewed as either:
    - Simplification of field theory, or
    - Extension of circuit theory
  - Approach: use intuition to replace "electrically large" elements with "distributed circuit elements" with known electrical characteristics. Typically convert 2- or 3-D problem into interconnected 1-D elements
  - Much simpler to compute: by basing analysis on known structures, can directly find V, I vs. position; no need to compute intermediary fields
  - Can often yield intuitive insight into circuit operation, since each element has (usually) relatively simple behavior
  - But: not rigorous. Relies on designer to:
    - Pick the right component to substitute in
    - If coupling between elements is important, designer must add that (or choose a component that has it built in)

Distributed Circuit Approaches

- How are these two approaches related?
  - Full field theory is rigorous, allows evaluation of new structures that are not understood
  - Transmission line theory elements are developed to mimic the E&M behavior of typical structures that have proven useful
  - ➢ Transmission line theory much more efficient, but may mislead
- In practice, not an either/or proposition
- Common approach:
  - Use transmission line approaches to find behavior for “standard” or “simple” parts of a circuit
  - Switch to full-field theory for tricky spots or areas for which the appropriate model isn’t clear
  - Once design is finalized, one last full-field analysis of the whole thing to avoid surprises. Much better to find out before the parts have been built…

Full Field Theory Approach

- Maxwell's equations, plus boundary conditions
- A quick recap of E&M:

Maxwell's equations:

$$\nabla \times E = -\frac{\partial}{\partial t} B$$

$$\nabla \times H = \frac{\partial}{\partial t} D + J$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot D = \rho$$

Constituitive relations:

$$B = \mu H$$

$$D = \varepsilon E$$

Remember what the terms all mean?