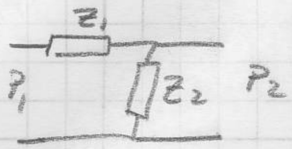
Problem 1



Part a) S-parameters:

$$S_{11} = \left. V_1^- / V_1^+ \right|_{V_2^+=0} \rightarrow \text{Port 2 terminated with } Z_0$$

$$\Rightarrow Z_{in} = Z_1 + (Z_2 // Z_0)$$

$$S_{11} = \frac{Z_1 + (Z_2//Z_0) - Z_0}{Z_1 + (Z_2//Z_0) + Z_0} = \frac{(Z_1 - Z_0)(Z_2 + Z_0) + Z_2 Z_0}{(Z_1 + Z_0)(Z_2 + Z_0) + Z_2 Z_0}$$

$$S_{22} = \left. V_2^- / V_2^+ \right|_{V_1^+=0} \rightarrow \text{Port 1 terminated with } Z_0$$

$$Z_{in} = Z_2 // (Z_1 + Z_0)$$

$$S_{22} = \frac{[Z_2 // (Z_1 + Z_0)] - Z_0}{[Z_2 // (Z_1 + Z_0)] + Z_0} = \frac{Z_2(Z_1 + Z_0) - Z_0(Z_1 + Z_2 + Z_0)}{Z_2(Z_1 + Z_0) + Z_0(Z_1 + Z_2 + Z_0)}$$

$$S_{21} = \left. V_2^- / V_1^+ \right|_{V_2^+=0} \qquad V_2 = V_2^- + V_2^+ \rightarrow V_2^+ = 0 \Rightarrow V_2 = V_2^-$$

Port 2 terminated with Z_0

$$S_{21} = \frac{V_2^-}{V_1^+} = V_2/V_1 \cdot V_1/V_1^+$$

$$V_2/V_1 = \frac{Z_2 // Z_0}{Z_1 + (Z_2//Z_0)} = \frac{Z_2 Z_0}{Z_1(Z_2 + Z_0) + Z_2 Z_0}$$

$$V_1/V_1^+ = \frac{V_1^+ + V_1^-}{V_1^+} = \frac{V_1^+(1 + S_{11})}{V_1^+} = 1 + S_{11}$$

$$S_{21} = [1 + S_{11}] \frac{Z_2 Z_0}{Z_1(Z_2 + Z_0) + Z_2 Z_0}$$

$S_{12} = S_{21}$ by reciprocity

Part b)

$Z_1 = 30\Omega$, $Z_2 = 75\Omega$, $Z_0 = 50\Omega$

$$S_{11} = \frac{(30-50)(75+50) + 75 \cdot 50}{(30+50)(75+50) + 75 \cdot 50} = 0.0909$$

$S_{22} = -0.1273$, $S_{21} = 0.5455$, $S_{12} = 0.5455$

$$S = \begin{bmatrix} 0.0909 & 0.5455 \\ 0.5455 & -0.1273 \end{bmatrix}$$

Part c)

Port 2 terminated in 50Ω $(= Z_0)$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}, \quad \Gamma = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0 \text{ for } Z_L = Z_0$$

$$\Rightarrow |\Gamma| = 0.0909$$

$\underline{VSWR = 1.2}$

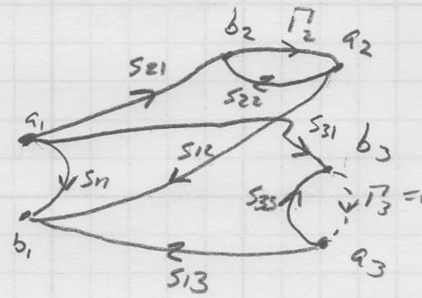
Problem 2

Part a) Reciprocal? Yes – $S = S^T$.

Part b) Input return loss:

$$RL = -20 \log_{10} |\Gamma| \quad (dB)$$

$\Gamma = b_1/a_1 \rightarrow$ Flow graph approach:



$$\Gamma_2 = \frac{0 - Z_0}{0 + Z_0} = -1 \quad (\text{short})$$

$$\Gamma_3 = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0 \quad (Z_0)$$

First-order loops: $\Gamma_2 S_{22}$

$T = b_1/a_1 \Rightarrow$ Paths: S_{11}, $S_{21}\Gamma_2 S_{12}$

$$T = b_1/a_1 = \frac{S_{11}[1 - \Gamma_2 S_{22}] + S_{21}\Gamma_2 S_{12}}{1 - \Gamma_2 S_{22}} = S_{11} + \frac{S_{21}\Gamma_2 S_{12}}{1 - \Gamma_2 S_{22}}$$

$$= 0.00536 - j0.0166 = 0.0174\angle -72^\circ$$

$$RL = -20 \log_{10} 0.0174 = -35.2\,dB$$

Part c)

Input impedance?

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \Longrightarrow \Gamma(Z_{in} + Z_0) = Z_{in} - Z_0$$

$$Z_{in} = Z_0 \frac{1+\Gamma}{1-\Gamma}$$

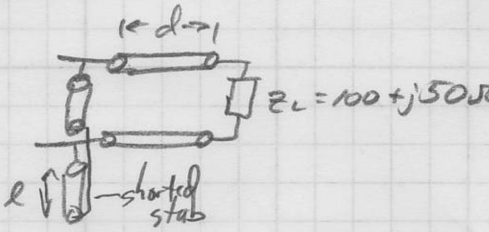
$$\Gamma = 0.0174\angle -72^\circ$$

$$\Longrightarrow Z_{in} = 50.5 - j\,1.68\,\Omega = 50.53\angle -1.9^\circ$$

Problem 3

Shunt-stub matching network:

All lines have $Z_0 = 50\,\Omega$



$$\Rightarrow z_L = \frac{Z_L}{Z_0} = 2 + j$$

$$y_L = 1/z_L = 0.4 - j0.2$$

Terminated line input impedance:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta d}{Z_0 + jZ_L \tan\beta d}$$

normalized:

$$z_{in} = \frac{z_L + j\tan\beta d}{1 + jz_L \tan\beta d}$$

$$y_{in} = \frac{1 + jz_L \tan\beta d}{z_L + j\tan\beta d}$$

At end of "d" line, $y_{in} = 1 + jb$

$$\Rightarrow 1 + jb = \frac{1 + j(2+j)\tan\beta d}{2+j + j\tan\beta d}$$

$$(1+jb)\left(2 + j(1 + \tan\beta d)\right) = 1 - \tan\beta d + j2\tan\beta d$$

Real part:

$$2 - b(1 + \tan\beta d) = 1 - \tan\beta d$$

Imag. part:

$$j2b + j(1 + \tan\beta d) = j2\tan\beta d$$

2 eq., 2 unknowns $(b, \tan\beta d)$

From Imag part: $b = \dfrac{\tan\beta d - 1}{2}$

From Imag. part:

$$2 - \frac{\tan\beta d - 1}{2}(1 + \tan\beta d) = 1 - \tan\beta d$$

$$5/2 - 1/2\tan^2\beta d = 1 - \tan\beta d$$

$$1/2\tan^2\beta d - \tan\beta d - 3/2 = 0$$

$$\tan\beta d = \frac{1 \pm \sqrt{1 + 4 \cdot 1/2 \cdot 3/2}}{1} = 1 \pm 2 = -1, 3.$$

Two solutions

$\tan\beta d = 3 \to b = 1$

$\tan\beta d = -1 \to b = -1$

$\tan \beta d = 3 \rightarrow \text{atan } 3 = \beta d = \frac{2\pi}{\lambda} d$

$\quad \hookrightarrow d/\lambda = \frac{\text{atan } 3}{2\pi} = 0.199$

$\tan \beta d = -1 \rightarrow d/\lambda = -0.125 \Longrightarrow d/\lambda = 0.375$ for length >0
(periodic w/ period $\lambda/2$)

stub design:

$y_{in} = 1 + jb \rightarrow$ so stub provides $-jb$

$y_{instub} = \frac{1 + j z_t \tan\beta l}{z_t + j \tan\beta l}$, z_t = stub termination impedance

short: $z_t = 0$

$y_{instub} = \frac{1}{j \tan\beta l}$

For $d = 0.199\lambda \rightarrow b = +1 \Rightarrow y_{instub} = -j \rightarrow \tan\beta l = 1$

$l = 0.125\lambda$

For $d = 0.375\lambda$, $b = -1 \rightarrow y_{instub} = +j \Rightarrow \tan\beta l = -1$

$l = -0.125 + 0.5 = 0.375\lambda$

Solutions:

$d_1 = 0.199\lambda$, $l_1 = 0.125\lambda$

OR

$d_2 = 0.375\lambda$, $l_2 = 0.375\lambda$

Bonus

match 150Ω to 50Ω with series line

$\rightarrow \lambda/4$ transformer

$$Z_1 = \sqrt{Z_0 \cdot Z_L} = \sqrt{50 \cdot 150} = 86.6\,\Omega$$

length $= \lambda/4$

For lines available,

$R = 0,\ G = 0 \rightarrow$ lossless

$L = 8\,nH/cm$

$C = ?$

$$Z_1 = 86.6\,\Omega = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} \quad \text{for } R, G = 0$$

$$\Rightarrow C = \frac{L}{Z_1^2} = 1.07\,pF/cm = C$$

Length of $\lambda/4$ transformer? (in physical units, e.g. cm)

Propagation constant $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$

$R, G = 0 \rightarrow \gamma = j\beta = j\omega\sqrt{LC}$

$$\beta = \omega\sqrt{LC} = 2\pi/\lambda$$

$$\lambda = \frac{2\pi}{\omega\sqrt{LC}} = \frac{1}{f\sqrt{LC}}$$

$f = 10\,GHz,\ L = 8\,nH/cm,\ C = 1.07\,pF/cm$

$\Rightarrow \lambda = 1.0825\,cm$

$\Rightarrow$ length of line $= \underline{\lambda/4 = 0.271\,cm}$