Defining Autonomy and Measuring its Levels
Goals, Uncertainties, Performance and
Robustness

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Introduction

Autonomous vehicles have certainly captured the imagination of everyone in recent years. The promise of reducing or even eliminating accidents via autonomy is very appealing indeed. Certainly, autonomy in engineered systems is not a new concept; and definitely it is not a new concept in organizations, in society, in biology. Note that automatic pilots for aircrafts and ships that increase the degree of autonomy of the system, have been operating very successfully for many years-the first autopilot for aircraft was introduced in 1912. These are examples of outcomes of the Quest for Autonomy, a pervasive theme in engineered systems through the centuries starting even earlier than Ktesibios’ waterclock with its feedback mechanism in the 3rd century BC and continuing strong today. It seems that we always wanted to build things that did more things by themselves. Adding to traditional control systems advanced sensing and incorporating decision making from areas such as AI is a way to increase substantially the level or degree of autonomy of a system. Control systems should be seen as the cornerstone of autonomous dynamic systems.

When people refer to autonomous systems they often mean different things. It is important to be more precise and agree upon a common definition such as: *If a system has the capacity to achieve a set of goals under a set of uncertainties in the system and its environment, by itself, without external intervention, then it will be called autonomous with respect to the set of goals under the set of uncertainties.* For the same set of goals, the larger the set of uncertainties the system can handle, the higher its degree of autonomy. The lower the needed external intervention by humans or other systems to achieve the goals under the uncertainties, the higher the degree of autonomy. So, the level of autonomy depends on both, a measure of the set of the goals that are being accomplished and a measure of the set of uncertainties present. Specifically, \( \{\text{Measure of the Set of Goals}\} \times \{\text{Measure of the Set of Uncertainties}\} = L \), the level of autonomy. This definition allows the comparison of the autonomy levels of different systems. These issues are discussed in detail in this paper.

The present paper focuses on measures of autonomy with emphasis on comparing levels or degrees of autonomy. The definition of autonomy used here was first presented in [1] and further discussed in [2] where the main ideas behind defining levels of autonomy were elaborated upon. It should be noted that the ideas of defining autonomy using sets of goals and uncertainties have appeared in the writings of the author, published in the open literature, much earlier; see for example [3]-[5]. Autonomy in engineering systems and its relation to intelligent behavior was discussed in the task force report [6]. Details of defining levels of autonomy were discussed in a paper draft [7] which was circulated and commented upon by colleagues. These ideas were also presented in a keynote address at the Mathworks Research Summit in early June 2019.
The purpose of the present paper is to define autonomy, describe concrete ways to talk about autonomy and levels of autonomy and provide quantitative relations.

We start the discussion with our definition of autonomy. The interested reader may want to read materials from [2]-[7] and consult the references therein which describe early research (in the late 1980s to mid 1990s) in combining control systems with intelligent methods from artificial intelligence and machine learning to design highly autonomous intelligent control systems. For additional discussion of autonomy and its levels see, for example, [8] where the definitions introduced correspond to the definitions in this paper in that they use task-specific goals to be achieved by the system and refer to needed outside intervention instead of uncertainties present. See also [9]-[13]. Note that a definition involving necessary outside interventions to achieve the goals is discussed later in this paper. It should be noted that, contrary to other definitions, our definition of autonomy does not involve descriptions of the means by which a specific level of autonomy is achieved, whether smart sensors or intelligent decision making are involved. We find it more useful to characterize autonomy using only the possible to achieve goals under given uncertainties and letting the specific means by which the level of autonomy is achieved to be used in characterizations of the system as smart, intelligent etc. In fact, as it was stated many times in our publications, “autonomy is the goal and intelligent means is one way to achieve it.” Higher autonomy typically involves higher intelligence.

Our definition of autonomous behavior provides a natural way to define levels or degrees of autonomy via simple quantitative relations, specifically, as it was mentioned above, \( \{\text{Measure of the Set of Goals}\} \times \{\text{Measure of the Set of Uncertainties}\} = L \), the level of autonomy. This easily leads to an intriguing and interesting relation, namely \( \{\text{Performance}\} \times \{\text{Robustness}\} = L \), the level of autonomy. Here Performance is a measure of the set of goals that can be achieved (and it may include stability) and Robustness (Resilience) is a measure of the set of uncertainties under which the goals are reached. Systems with higher performance and/or higher Robustness/Resilience have higher degree of autonomy. These issues are discussed in detail later in the present paper.

Entropy can also be used as a general measure of the set of uncertainties. Entropy in autonomy is discussed at the end of this paper.

An additional interesting measure is the degree of external intervention needed to achieve the set of goals. The higher the needed external intervention the lower the level of uncertainties under which the goals can be achieved; that is there exists an inversely proportional relation between the level of needed external intervention and the level of uncertainties or robustness under which the system operates when achieving the set of goals.

Examples are used throughout this paper to illustrate the concepts including a glimpse of how these new definitions and relations may be applied to the 5 autonomous vehicle levels used in the self-driving car literature and industry.
Definitions and Measures

Etymology of the word autonomy: The term autonomy originated in Ancient Greek: αὐτονομία (autonomia), from αὐτόνομος (autonomos), which comes from αὐτό (auto) "self" and νόμος (nomos) "law", hence when combined it is understood to mean one who gives oneself his/her own law. Autonomous means having the capability and authority for self-governance.

Autonomy goals: A system exhibits autonomous behavior only while achieving a goal or a set of goals. That is, autonomy without clearly identified goals, autonomy for the sake of autonomy is not interesting, if we want to build useful engineering systems. Autonomy without goals is as vague a concept as claiming that something is optimal without specifying a measure, such as a cost to be minimized. For example, a goal of an autonomous train could be to move passengers safely from station to station following a time schedule with some probability; the goal of a speed cruise control of an automobile is to control the car so to maintain approximately constant speed.

Every autonomous system is a control system: An autonomous system always has a set of goals to be achieved and a control mechanism to achieve them. This implies that every autonomous system is a control system. Here the term “control system” is used in a most general sense, in which control (a decision mechanism typically using sensor measurements and feedback together with ways to implement these decisions via actuators) is used to make the system (a very general collection of processes) attain desirable goals.

As it was mentioned above, the word control in autonomous control has a more general meaning than in conventional control; in fact, it is closer to the way the term control is used in every-day language; see [4]. To illustrate, in a rolling steel mill, while conventional controllers may include the speed (rpm) regulators of the steel rollers, in the autonomous control framework one may include in addition, fault diagnosis and alarm systems; and perhaps the problem of deciding on the set points of the regulators, that are based on the sequence of orders processed, selected based on economic decisions, maintenance schedules, availability of machines etc. All these factors have to be considered as they play a role in controlling the whole production process, which is really the overall goal. Note that in order to increase autonomy it is typical to implement several layers/levels of automation. Local controllers are often referred to as level 1 automation, set points assignment as level 2, and so on.

System and its environment: As it is typically done in the field of control systems, it is useful to think of a system to be controlled as being surrounded by a boundary separating it from its environment. The system acts upon its environment through its outputs and receives inputs in the form of disturbances or additional information. What the system includes within its boundary, expressed via the particular system model used, depends of course on the goals and the characteristics/properties used to achieve its goals.

Goals and Uncertainties: In addition to the set of goals to be attained the other central component of autonomy is the set of uncertainties. For example, in the above cruise control example, the speed needs to be maintained (goal) under varying external conditions such as road incline, condition of road surface, wind gusts, as well as internal varying vehicle conditions such
as hot or cold engine and age of the car (uncertainties). Clearly the uncertainties of interest in an autonomous system are the ones that affect the accomplishments of the goals.

So, autonomy is the ability of a system to achieve a set of goals under uncertainties in the system and its environment. Autonomy exists only with respect to a set of goals and it is of value when there are uncertainties. If there were no uncertainties, we could program the system ahead of time, in which case a macro-command would be adequate. In control system theory if we had complete knowledge of the system to be controlled and of the external disturbances then we could only use open loop control and the control problem would have been rather straightforward. Uncertainties however are always present in different degrees. For example, in the above case of the train moving on fixed rails from station to station, as in an airport terminal, there are reasonable guarantees that no passenger will cross the rails and there will be an unobstructed path for the train and so the uncertainties are rather limited and are primarily caused by the flow of passengers in and out the train at each station. This problem is manageable and currently such automated trains are operating successfully in many airports around the world. Compare this with a car moving from point A to point B. Even if we assume that the car stays in the same lane, the problem is much harder because there are uncertainties such as traffic lights, other cars changing lanes without warning, pedestrians crossing unexpectedly, the weather that affects braking distance and so on. Because of the increased uncertainties designing autonomous cars to operate in a city is much harder than designing autonomous trains to operate in an airport terminal. It should be noted that significant successes have been achieved in airplane automatic pilot systems that are being used thousands of times daily which maintain direction, speed and altitude under unexpected disturbances such as gusts of wind and air pockets.

In view of the above discussion we introduce the following definition which captures the fact that autonomy should always be considered in terms of goals attained under uncertainty.

Given a system S, let G be a set of goals to be attained under a set of uncertainties U.

**Definition:** A system S is autonomous with respect to the set of goals G under the set of uncertainties U if the system S is capable of achieving all goals in G in the presence of all uncertainties in U, by itself, without external intervention.

The set of uncertainties U is associated with the set of goals G. It is implied that the uncertainties considered in the above definition are the ones that are relevant to the goals considered. For example, for the goal of stability certain uncertainty in the parameters may be relevant, but different set of uncertainties may be relevant when the goal is tracking. Other uncertainties which are irrelevant to the goals of interest do of course exist; for example, in designing the autopilot of an aircraft we do not consider the interior design unless it has implications on the weight of the aircraft.

It is assumed that the system S may perform these functions autonomously over a significant time horizon; that is, this is a repeatable function the system is capable of, over extended periods of time.
It is possible to have as a goal to control the system so that some property is attained with certain probability. For example, the goal could be to attain asymptotic stability with a probability of 95%. So, the above definition captures the realistic scenarios of achieving goals with certain likelihood.

The above definition of autonomy should and does apply to organizations and natural systems as well. For example, in an organization, a team led by a manager accomplishes a set of tasks under uncertainties such as personnel absences and equipment breakdowns, independently without intervention from a general manager. A bacterium may be able to reach a light under normal circumstances but needs external help to remove unexpected obstacles in its path.

**Degrees or Levels of Autonomy**

It is of interest to compare the levels of autonomy in systems. Assume that a given system is autonomous with respect to a set of goals under a set of uncertainties. If another system can achieve the same goals under higher uncertainties (under a larger set of uncertainties) then clearly the second system has higher autonomy. Similarly, if more goals can be achieved under the same set of uncertainties then the system has higher autonomy.

The autonomy level of a system can be manipulated and increased by adding feedback control, adaptation, learning, planning, failure detection and reconfiguration capabilities, which in effect increase the level of uncertainties the system can cope with autonomously.

A fixed feedback control system has low degree of autonomy because it can achieve the stability goals under rather restricted parameter variations and external disturbances. When there are more substantial parameter changes then one could use methods from adaptive control to achieve stability. Such adaptive control system has higher degree of autonomy due to greater uncertainty in the parameters it can handle.

The degree of autonomy can be interpreted as the size of an operating region (operating sphere) defined by a set of parameters within which the system acts on its own in a safe manner towards the goal. In the example of the car speed control, a typical cruise control system can keep the car speed at acceptable levels only when the road is not too steep. And such control system has certain degree of autonomy as it acts appropriately within its operating region, which is specified by the initial design of the system. We could build cruise control systems with larger operating regions satisfying the goal of keeping the speed at a preset desired level. One way to achieve this is to anticipate, via perhaps a vision system an upcoming steep grade and prepare for it by shifting gears or accelerating slightly, which is exactly what human drivers typically do. We could also have car speed control systems that may attain additional goals thus increasing even more their operating regions. For example, we could add in a car a control system that maintains the same speed as the car in the front (these are called ACC-advanced cruise control systems), and in addition it adjusts the distance between the cars depending on the speed, for safety reasons. It is clear that these two control systems, taken together can satisfy a set of goals under quite diverse conditions. Clearly such system has higher degree of autonomy.
For given set of goals, the degree of autonomy may be quantified by characterizing the safe operating region within which the system acts appropriately. This region in control systems is sometimes referred to as region (ball, sphere) of uncertainty and it is characterized by certain norm measures, when of course normed spaces are appropriate. Control systems that act appropriately in these uncertainty regions are called robust with respect to these uncertainty regions and with respect to goals such as stability (typically Lyapunov asymptotic stability) or performance.

Note that the same system may be autonomous or not depending on the stated goals and the uncertainties present. Furthermore, a non-autonomous system may have several autonomous functions. For example, in cars, the cruise control, the ABS, ACC, lane preserving, etc., offer autonomous functionalities and for each one of these subsystems the set of goals and the uncertainties could be identified.

Autonomous systems deal with uncertainties primarily using sensors, but also, for example, using prior knowledge and machine learning, to improve their knowledge of the processes to be controlled and also of the outside environmental influences, so to be able to achieve the goals by applying effective decision-making methods. Intervention (human or via a controller) reduces uncertainties the system has to deal with autonomously. Successful control actions, by engineered systems or human intervention, reduce the set of uncertainties that impact the goals and must be dealt with autonomously. Human intervention or adaptive/learning controllers may provide information via, for example, cognitive abilities, data bases, prior experience that reduce the uncertainties, and lead to a smaller set of uncertainties that need to be dealt with autonomously.

Measuring the degree of autonomy is non-trivial. It is perhaps straightforward to compare systems that have the same sets of goals but different uncertainties. It was pointed out above that an adaptive control system has higher degree of autonomy than a fixed feedback controller because it can handle greater parameter uncertainty in achieving stabilization (the common goal). When the goals are different as well, then the problem of measuring degrees of autonomy and comparing autonomous systems becomes more complex.

The automotive industry currently uses a useful, descriptive classification to distinguish levels of autonomy. There is a SAE scale of 5 levels (plus a zero level) with level 5 used for full autonomy. Similarly, the AFRL Autonomy Framework is used in the UAV area, where a scale of 10 levels (plus a zero level) is being used with level 10 used for full autonomy.

**Relations**

The above discussed relationships that help us characterize different degrees of autonomy may be captured by the following very simple relations:

Level or degree of Autonomy = \{Measure of the Set of Goals G\} x
\{Measure of the Set of Uncertainties U under which the goals in G are attained\}
Let \( M_G \) be a measure of the set of goals \( G \) and \( M_U \) be a measure of the set of uncertainties \( U \) and \( L \) be a measure of the level of autonomy of the system.

Then \( L = M_G \times M_U \)

\( L \), the level or degree of autonomy, depends on both, the measure of the set of uncertainties and the measure of the set of the goals that can be accomplished.

The measure of the set of goals should reflect the importance, complexity and number of goals. Importance may depend on existing priorities – tracking quickly within a few seconds may be a higher priority than tracking asymptotically and in this case the level of autonomy with respect to the finite tracking is smaller if only asymptotic tracking may be achieved. Similarly, the measure of the set of uncertainties should reflect the size, frequency and number of uncertainties.

For a given level of autonomy \( L \), when \( M_U \) decreases, \( M_G \) increases that is under reduced uncertainty more goals can be achieved by the system. When \( M_U \) goes up, \( M_G \) goes down that is, under increased uncertainty fewer goals can be achieved.

When the goal is just stability and the uncertainties are small, that is \( M_G \) and \( M_U \) are small then the level of autonomy \( L \) is low. This is the case for example when stabilization can be achieved via a fixed feedback controller. When stabilization can be achieved under higher uncertainties, which is the case for example when adaptive control is used to stabilize a system, the level of autonomy \( L \) is higher. To increase \( L \), when there is a fixed set of goals, one needs to increase uncertainties under which the system is capable of achieving the goals.

Appropriate controllers in effect increase the size of the set of uncertainties relevant to the goals that can be accomplished autonomously and increase the system’s level of autonomy. Note that these controllers are modifying the system. Uncertainties that can be dealt with autonomously may be increased, for example using adaptation and learning, or human intervention, where extra sensors, cognitive abilities, past experience effectively increases the set of uncertainties the system can cope with autonomously. For example, consider the case when a driver intervenes and assumes certain functions to help the vehicle cope with uncertain situations.

Clearly, by introducing restrictions on the uncertainties in autonomous vehicles (e.g. adding structure - staying in the same lane, using rails, assuming good weather etc.) more goals can be achieved.

More goals can be achieved by adding additional controllers. For example, assume that a given system is stabilized via a feedback controller, which operates successfully over a set of uncertainties. If a tracking controller is added the goals that can be achieved increase; however, the set of uncertainties that can be dealt with autonomously while tracking may be reduced compared to the stabilization case.

Given a system, if there are no uncertainties at all, a much-enhanced set of goals may be achieved with appropriate controllers. For example, we could use open loop control to cancel all existing dynamics and introduce any new desired dynamics. However, note that when a system
is run open loop, uncertainties in plant parameters and disturbances could deny the ability to achieve control goals, such as stability.

**Given a system is there a maximum L?**

The answer is affirmative. For a given system there is a maximum set of goals that can be achieved. For example, the attainable goals for a self-driving vehicle do not include the ability to fly – at least not yet.

Considering this maximum set of goals, consider the set of uncertainties that affect those goals and then consider the largest set of uncertainties under which this set of goals can be attained.

To find the maximum autonomy level of a system, consider the measures for the set of goals and the set of uncertainties under which these goals are achieved and then maximize their product by varying the sets of goals and for each set of goals selecting the corresponding set of uncertainties that have the maximum measure.

**External Intervention**

Autonomy may also be defined in terms of needed, necessary outside intervention to achieve the goals instead of in terms of a set of disturbances. An equivalent definition of autonomy is:

**A system is autonomous with respect to a set of goals G under a set of outside interventions I (by humans or engineered systems), when the system can achieve all the goals in G, assisted by just the interventions I.**

If the goals can be achieved under a smaller set of outside (human and otherwise) interventions, then the system may cope with higher uncertainties and has higher autonomy; if more goals can be achieved under the same set of interventions or the same set of uncertainties then the system has higher autonomy.

The lower the needed intervention to accomplish the goals, the higher the level of autonomy. The uncertainty the system can cope with while achieving its goals, is inversely proportional to intervention necessary to achieve the same goals.

\[
\{\text{Measure of the Set of Interventions I under which the goals in G are attained}\} \times \{\text{Measure of the Set of Uncertainties U under which the goals in G are attained}\} = M_I \times M_U = 1
\]

These relationships may be captured via a simple relation:

Level or degree of Autonomy = \{Measure of the Set of Goals G\} / \{Measure of the Set of Interventions I under which the goals in G are attained\}
Let $L$ be a constant that corresponds to the level of autonomy. Let $M_G$ be a measure of the set of goals $G$ and let $M_I$ be a measure of the set of needed interventions $I$. Then

$$L = \frac{M_G}{M_I}$$

Note that here it was assumed that $M_U \times M_I = 1$. That is, $M_I$ the measure of the set of needed interventions may be taken to be inversely proportional to $M_U$ the measure of the set of uncertainties $U$.

As an example, consider a home thermostat. A simple thermostat can achieve the goal of thermal comfort with minimum energy use, with user interventions to change the set-point when residents leave or return to home, sleep, etc. A smart thermostat could achieve the goal without this level of human intervention, relying on occupancy sensors, models of thermal comfort at night versus daytime, etc. The smart thermostat has higher level of autonomy as it can achieve the goal with lower user intervention.

In certain cases, human intervention is needed to take care of a subset of the existing disturbances thus eliminating them from the set of uncertainties the system needs to cope with. Such intervention allows the system to attain the goals autonomously, under the now reduced set of disturbances. For example, the cruise control in a car that maintains the car’s speed constant may be not be able to perform if the road incline is very steep. The driver may intervene using say look ahead control policies to cope with these large size uncertainties of the road incline and so reducing the set of incline uncertainties the system needs to deal with autonomously.

**Performance and Robustness**

*Performance* may be taken to be a measure of the set of goals $G$ achieved by the system. A performance level is assigned that captures the number of goals, their difficulty and importance. It should be noted that the term Performance here has a more general meaning than in the Controls literature, where typically it does not include stability. A level of Performance is accomplished under a level of *Robustness* which corresponds to the level of uncertainty under which the goals are achieved. For fixed performance level, higher level of robustness implies higher autonomy. Also, for fixed robustness level, higher level of performance implies higher autonomy level. For fixed autonomy level, higher performance leads to lower robustness and higher robustness leads to lower performance.

Level of autonomy $L = \{\text{Performance}\} \times \{\text{Robustness}\}$

Performance $P$ is a particular measure of the set of goals $G$, $M_G$. Robustness $R$ is a particular measure of the set of uncertainties $U$, $M_U$. For $P = M_G$ and $R = M_U$,

$$L = P \times R$$

For fixed level of autonomy $L$ when Performance increases Robustness must be reduced. This brings up interesting issues regarding fundamental limitations.
Robustness $R$ which is a measure of the uncertainties the system can cope with is inversely proportional to $M_I$, the level of needed outside intervention.

$$R \times M_I = 1$$

**Summary of Measures and Relations**

Let $M_G$ be a measure of the set of goals $G$. Let $M_U$ be a measure of the set of uncertainties $U$. If $L$ is the level of autonomy of the system then

$$L = M_G \times M_U$$

Performance $P$ can be seen as an $M_G$. Robustness $R$ can be seen as an $M_U$. Then for $P = M_G$ and $R = M_U$ the above relation becomes

$$L = P \times R$$

Let $M_I$ be a measure of the set of needed interventions $I$. Then for $M_I \times M_U = 1$ the above relation becomes

$$L = M_G / M_I$$

In view of the relation between measures of Uncertainty and Intervention, namely

$$M_I \times M_U = 1$$

and the fact that measure of Robustness $R = M_U$ we have that

$$M_U = R = 1/M_I$$

that is the smaller the needed intervention the higher the robustness of the system.

**Entropy**

*Using Entropy to compare autonomous systems that achieve the same set of goals:* Entropy is a measure of uncertainty. If two systems accomplish the same goals, the system with higher Entropy has a higher level of autonomy since the goals are achieved under greater uncertainties.

**For the same goals, higher Entropy implies higher levels of autonomy.**

*Using Entropy to compare autonomous systems with varying sets of goals:* Entropy measures uncertainty. Reduced entropy means reduced uncertainty that implies an increase of the set of goals possible, that is a higher level of autonomy.

As Entropy decreases the set of goals that may be achieved increases. When Entropy is epsilon or zero, a very large set of goals may be accomplished – restricted only by the system’s characteristics, its dynamics and structure.

As Entropy increases the set of goals that maybe achieved decreases. When Entropy is very large the set of goals that can be achieved becomes very small – epsilon size or zero.
Let $M_G$ be a measure of the set of goals $G$ and $M_U$ be a measure of the set of uncertainties $U$. We have seen that $L = M_G \times M_U$

Entropy can be taken to be the measure for the uncertainties. For Entropy $H = M_U \times L = M_G \times H$

**Humans in the Loop and Adaptive Autonomy**

When one considers humans collaborating with engineered systems, then the overall system that includes humans in the loop may be considered autonomous with respect to a large set of goals and under a large class of uncertainties, that is having a high level of autonomy. Depending on the role of the humans in the loop and the level of control authority humans exert, the remaining system will have different degrees or levels of autonomy. So, in an automobile, if for example the goal is to keep the vehicle inside a lane while travelling with constant speed, the system may consist of the vehicle and the driver where the system attains its goals in the presence of uncertainties/disturbances, such as gusts of wind and road inclines. The driver together with the automobile’s control systems provide the correct steering and gas pedal commands so the vehicle maintains its course within a lane and at certain (approximately) constant speed in the presence of uncertainties/disturbances, such as gusts of wind and road inclines. If one considers the controller to consist of just the control systems of the car without the driver, then the system, the car, has a lower degree of autonomy, meaning that it may need extra help from humans or other systems to attain the required level of autonomy.

Humans or other systems may insert themselves at different levels of a functional hierarchy (that correspond to different levels of autonomy) used to describe the operation of autonomous intelligent systems [2-6], and take over control functions. For example, humans may insert themselves to take over planning, failure detection and identification, reconfiguration or learning functions. Or they may insert themselves to take over lower control functions e.g. a driver may want to take over the functions of the ABS system to perform the braking pumping action on his own. Such *adaptive autonomy*, where the authority the human operator exercises may vary, appears to be a very promising direction in autonomous systems research. The level of authority of the human operator may vary and the changes may be initiated not only by the human operator, but also by the vehicle if it detects driver errors or lowering of the driver’s alertness.

**Optimization and Autonomy definitions analogy.**

Optimal with respect to a set of goals (a performance measure) subject to a set of constraints. Autonomous with respect to a set of goals (a measure) subject to a set of uncertainties

The constraints restrict the set of possible solutions. The constraints may be severe enough for a feasible solution not to exist (set of constraints be infeasible) in which case no optimal solution exists (in fact no solution exists at all). The uncertainties restrict the set of possible control policies that achieve the goals. The uncertainties may be large enough for no control policies to exist that achieve the goals (the set of uncertainties render the problem infeasible) in which case no control policy exists that make the system autonomous with respect to the given set of goals.
References


APPENDIX A

Consider the 0 to 5 levels of autonomy in vehicles. We have

\[ M_G \times M_U = L \]

and

\[ M_G / M_I = L \]

Let the scale for \( M_G \) be 0-10 and the scale for \( M_U \) be 0-10. Then the range of level of autonomy will be 0-100.

Assume that the goals are the same - to drive at the level of a human driver under any normal road conditions. We shall take \( M_G \) to be equal to 10 across all levels.

At level 5 we calibrate \( M_U \) to be 10, which implies that all goals are achieved under maximum uncertainties. At level 0 we calibrate \( M_U \) to be 0.1, which implies that all goals are achieved under minimum or no uncertainties.

In summary

At level 0, \( M_U = .1 \)
At level 1, \( M_U = 2 \)
At level 2, \( M_U = 4 \)
At level 3, \( M_U = 6 \)
At level 4, \( M_U = 8 \)
At level 5, \( M_U = 10 \)

The levels of autonomy then will be

At level 0, \( \{M_G = 10\} \times \{M_U = .1\} = 1 = A_L \) the level of autonomy.
At level 1, \( \{M_G = 10\} \times \{M_U = 2\} = 20 = A_L \)
At level 2, \( \{M_G = 10\} \times \{M_U = 4\} = 40 = A_L \)
At level 3, \( \{M_G = 10\} \times \{M_U = 6\} = 60 = A_L \)
At level 4, \( \{M_G = 10\} \times \{M_U = 8\} = 80 = A_L \)
At level 5, \( \{M_G = 10\} \times \{M_U = 10\} = 100 = A_L \)

Instead of Uncertainty consider now a measure of required Intervention for the goals to be achieved.

Let the scale for \( M_G \) be 0-10 and the scale for \( M_I \) be 0-10. Then the range of level of autonomy will be 0-100.
At level 5 we calibrate $M_I$ to be $1/10=0.1$, which implies that all goals are achieved under minimum or no intervention. At level 0 we calibrate $M_I$ to be $1/0.1=10$, which implies that all goals are achieved only under maximum intervention.

In summary

At level 0, $M_I = 1/0.1 = 10$
At level 1, $M_I = 1/2$
At level 2, $M_I = 1/4$
At level 3, $M_I = 1/6$
At level 4, $M_I = 1/8$
At level 5, $M_I = 1/10 = 0.1$

The levels of autonomy then will be

At level 0, $\{M_G = 10\} / \{M_I = 1/0.1=10\} = 1 = A_L$ the level of autonomy.
At level 1, $\{M_G = 10\} / \{M_I = 1/2\} = 20 = A_L$
At level 2, $\{M_G = 10\} / \{M_I = 1/4\} = 40 = A_L$
At level 3, $\{M_G = 10\} / \{M_I = 1/6\} = 60 = A_L$
At level 4, $\{M_G = 10\} / \{M_I = 1/8\} = 80 = A_L$
At level 5, $\{M_G = 10\} / \{M_I = 1/10=0.1\} = 100 = A_L$

We could have taken

At level 0, $M_I = 1/0.1 = 10$
At level 1, $M_I = 8$
At level 2, $M_I = 6$
At level 3, $M_I = 4$
At level 4, $M_I = 2$
At level 5, $M_I = 1/10 = 0.1$

In that case

The levels of autonomy then will be

At level 0, $\{M_G = 10\} / \{M_I = 1/0.1=10\} = 1 = A_L$ the level of autonomy.
At level 1, $\{M_G = 10\} / \{M_I = 8\} = 10/8 = A_L$
At level 2, $\{M_G = 10\} / \{M_I = 6\} = 10/6 = A_L$
At level 3, $\{M_G = 10\} / \{M_I = 4\} = 10/4= A_L$
At level 4, $\{M_G = 10\} / \{M_I = 2\} = 10/2= A_L$
At level 5, $\{M_G = 10\} / \{M_I = 1/10=0.1\} = 100 = A_L$

The constant then takes on different values from the case when Uncertainties are considered.
APPENDIX B

Complete list of author’s publications that are related to autonomy:


