Addendum to “Bonus Vetus OLS”  
(Baier and Bergstrand, or B-B, 2007)

In “Bonus Vetus OLS,” the MR (linear) approximations are derived under the assumption of symmetric bilateral trade costs (SBTC), or $t_{ij} = t_{ji}$ (as in A-vW, 2003). As an approximation method, the comparative-static trade effects of trade-cost changes should naturally be biased relative to the “true” model (the A-vW system), as we show in the paper.

However, it is possible that in a world with asymmetric bilateral trade costs (or ABTC, $t_{ij} \neq t_{ji}$) that, on average, the bias of the comparative-static trade effects of trade-cost changes using the approximation method are considerably smaller than the bias of the comparative statics using the A-vW approach (since the A-vW approach assumes SBTC). In fact, systematic Monte Carlo evidence of this is provided in Table 2 in Bergstrand, Egger and Larch (October 2007 working paper at www.nd.edu/~jbergstr/working_papers.html). This provides another motivation for the approximation method in our paper.

In this addendum, we demonstrate why, in a world with ABTC, the MR approximation terms provide good approximations to the BV approximation terms generated under SBTC in B-B. In other words, we solve here for MR approximations allowing ABTC.

We start with A-vW’s (2003) 2N equations (10) and (11) for MR under asymmetry:

\[
\Pi_i^{1-\sigma} = \sum_{j=1}^{N} \left( t_{ij} / P_j \right)^{1-\sigma} \theta_j \tag{A-vW, 10}
\]

\[
P_j^{1-\sigma} = \sum_{i=1}^{N} \left( t_{ij} / \Pi_i \right)^{1-\sigma} \theta_i \tag{A-vW, 11}
\]

Applying the first-order log-linear Taylor-series expansion in B-B, we can derive

\[
(1 - \sigma) \ln \tilde{\Pi}_i = -(1 - \sigma) \sum_{j=1}^{N} \theta_j \ln \tilde{P}_j + (1 - \sigma) \sum_{j=1}^{N} \theta_j \ln \tilde{t}_{ij} \tag{1}
\]

and
\[(1-\sigma) \ln \tilde{P}_j = -(1-\sigma) \sum_{i=1}^{N} \theta_i \ln \tilde{\Pi}_i + (1-\sigma) \sum_{i=1}^{N} \theta_i \ln \tilde{t}_{ij}\]  

where notation is defined in B-B. We can easily divide both equations by (1-\sigma) to eliminate that term.

Henceforth, for ease of notation, we will use the terms \( P_i \equiv (1-\sigma) \ln \tilde{P}_i, \Pi_i \equiv (1-\sigma) \ln \tilde{\Pi}_i, \) and \( t_{ij} \equiv (1-\sigma) \ln \tilde{t}_{ij}. \) NOTE: We have re-defined \( P_i, \Pi_i, \) and \( t_{ij} \) (vis-a-vis A-vW).

Because of the complexity of allowing ABTC and the matrix inversion needed, we assume a three-country world \( (i = 1, 2, 3). \) It will be useful now to specify, in this case, what B-B would derive for the MR approximation in the case of SBTC. For illustration, \( P_2 \) and \( P_3 \) would be:

\[
P_2 = \theta_1 t_{12} + \theta_2 t_{22} + \theta_3 t_{32} = \theta_1 t_{21} + \theta_2 t_{22} + \theta_3 t_{23} \]  

\[
P_3 = \theta_1 t_{13} + \theta_2 t_{23} + \theta_3 t_{33} = \theta_1 t_{31} + \theta_2 t_{32} + \theta_3 t_{33} \]

Defining \( t_2 \equiv \theta_1 t_{12} + \theta_2 t_{22} + \theta_3 t_{32}, t_3^* \equiv \theta_1 t_{21} + \theta_2 t_{22} + \theta_3 t_{23}, t_3 \equiv \theta_1 t_{13} + \theta_2 t_{23} + \theta_3 t_{33}, t_3^* \equiv \theta_1 t_{31} + \theta_2 t_{32} + \theta_3 t_{33} \)

for simplicity, we have:

\[
P_2 = t_2 = t_2^* \]

\[
P_3 = t_3 = t_3^* \]

Analogously, we define \( t_1 = \theta_1 t_{11} + \theta_2 t_{21} + \theta_3 t_{31} \) and \( t_1^* = \theta_1 t_{11} + \theta_2 t_{12} + \theta_3 t_{13}. \)

In a three-country world, the system of 2N equations simplifies to:

\[
\begin{bmatrix}
\theta_1 & \theta_2 & \theta_3 & 1 & 0 & 0 \\
\theta_1 & \theta_2 & \theta_3 & 0 & 1 & 0 \\
\theta_1 & \theta_2 & \theta_3 & 0 & 0 & 1 \\
1 & 0 & 0 & \theta_1 & \theta_2 & \theta_3 \\
0 & 1 & 0 & \theta_1 & \theta_2 & \theta_3 \\
0 & 0 & 1 & \theta_1 & \theta_2 & \theta_3
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
P_3 \end{bmatrix}
= \begin{bmatrix}
t_1^* \\
t_2^* \\
t_3^*
\end{bmatrix}
\begin{bmatrix}
\Pi_1 \\
\Pi_2 \\
\Pi_3
\end{bmatrix}
\]

(5)

To solve the system of equations for \( P_1, P_2, P_3, \Pi_1, \Pi_2, \) and \( \Pi_3, \) we need to invert the LHS 6x6 matrix.
However, the determinant of this matrix is zero, implying a redundant equation (because \( \theta_1 + \theta_2 + \theta_3 = 1 \) is imposed). We set \( P_1 (=1) \) as the numeraire and also eliminate one equation, \( \theta_1 P_1 + \theta_2 P_2 + \theta_3 P_3 + \Pi_1 = t_1^* \). This leaves the following system of 5 equations in 5 unknowns \( (P_2, P_3, \Pi_1, \Pi_2, \Pi_3) \):

\[
\begin{pmatrix}
\theta_2 & \theta_3 & 0 & 1 & 0 \\
\theta_2 & \theta_3 & 0 & 0 & 1 \\
0 & 0 & \theta_1 & \theta_2 & \theta_3 \\
1 & 0 & \theta_1 & \theta_2 & \theta_3 \\
0 & 1 & \theta_1 & \theta_2 & \theta_3 \\
\end{pmatrix}
\begin{pmatrix}
P_2 \\
P_3 \\
\Pi_1 \\
\Pi_2 \\
\Pi_3 \\
\end{pmatrix}
=
\begin{pmatrix}
t_2^* \\
t_3^* \\
t_1 \\
t_2 \\
t_3 \\
\end{pmatrix}
\tag{6}
\]

Since the determinant of the LHS 5x5 matrix is nonzero (and equal to \( \theta_1 \)), we can invert this matrix (matrix algebraic derivations of inversion available on request). Consequently, we find:

\[
\begin{pmatrix}
P_2 \\
P_3 \\
\Pi_1 \\
\Pi_2 \\
\Pi_3 \\
\end{pmatrix}
=
\begin{pmatrix}
0 & 0 & -1 & 1 & 0 \\
0 & 0 & -1 & 0 & 1 \\
-\theta_2 / \theta_1 & -\theta_3 / \theta_1 & 2 - \theta_1 & \theta_2 (\theta_2 + \theta_3) / \theta_1 & \theta_3 (\theta_2 + \theta_3) / \theta_1 \\
1 & 0 & \theta_2 + \theta_3 & -\theta_2 & -\theta_3 \\
0 & 1 & \theta_2 + \theta_3 & -\theta_2 & -\theta_3 \\
\end{pmatrix}
\begin{pmatrix}
t_2^* \\
t_3^* \\
t_1 \\
t_2 \\
t_3 \\
\end{pmatrix}
\tag{7}
\]

We can now examine the factors influencing each price term; since good 1 is the numeraire, we ignore \( P_1 \) and \( \Pi_1 \). We have:

\[
P_2 = t_2 - t_1 = \sum_{i=1}^{3} \theta_i t_{ij} - \sum_{i=1}^{3} \theta_i t_{i1} \tag{8a}
\]

\[
P_3 = t_3 - t_1 = \sum_{i=1}^{3} \theta_i t_{ij} - \sum_{i=1}^{3} \theta_i t_{i1} \tag{8b}
\]

\[
\Pi_2 = t_2^* + \theta_2 (t_1 - t_2) + \theta_3 (t_1 - t_3) = \sum_{j=1}^{3} \theta_j t_{2j} + \sum_{j=1}^{3} \theta_j t_{1j} - \sum_{j=1}^{N} \sum_{j=1}^{N} \theta_j \theta_j t_{ij} \tag{8c}
\]

\[
\Pi_3 = t_3^* + \theta_2 (t_1 - t_2) + \theta_3 (t_1 - t_3) = \sum_{j=1}^{3} \theta_j t_{3j} + \sum_{j=1}^{3} \theta_j t_{1j} - \sum_{j=1}^{N} \sum_{j=1}^{N} \theta_j \theta_j t_{ij} \tag{8d}
\]

It is clear that this will generalize to an N-country setting, such that for any countries \( i \) and \( j \) (not equal to 1,
the numeraire):

\[ P_j = \sum_{i=1}^{N} \theta_{ij} t_{ij} - \sum_{i=1}^{N} \theta_{j} t_{i1}, j = 2, \ldots, N \]  \hspace{1cm} (9a)

\[ \Pi_i = \sum_{j=1}^{N} \theta_{j} t_{ij} + \sum_{i=1}^{N} \theta_{i} t_{i1} - \sum_{i=1}^{N} \sum_{j=1}^{N} \theta_{ij} t_{ij}, i = 2, \ldots, N \]  \hspace{1cm} (9b)

First, note that, in a setting with \( N \) countries, the second RHS term in equation (9a) will be constant across all \( P_j (j = 2, \ldots, N) \); it reflects that country 1 is the numeraire. Hence, variation in \( P_j \) will be driven by the first RHS term in (9a), which is precisely the (log-linear) approximation used in B-B for the exporting country \( i \). This is confirmed by examining equations (8a) and (8b).

Second, note that, in a setting with \( N \) countries, the second and third RHS terms in equation (9b) will be constant across all \( \Pi_i (i = 2, \ldots, N) \). The second RHS term reflects that country 1 is the numeraire; the third RHS term reflects that an “inward” price is the numeraire. Hence, variation in \( \Pi_i \) will be driven by the first RHS term in (9b), which is precisely the (log-linear) approximation used in B-B for importing country \( j \). This is confirmed by examining equations (8c) and 8d).

Finally, we can rewrite the above equations using the original notation:

\[ \ln \Pi_i = \sum_{j=1}^{N} \theta_{ij} \ln t_{ij} + \sum_{i=1}^{N} \theta_{i} \ln t_{i1} - \sum_{i=1}^{N} \sum_{j=1}^{N} \theta_{ij} \ln t_{ij}, i = 2, \ldots, N \]  \hspace{1cm} (10a)

\[ \ln P_j = \sum_{i=1}^{N} \theta_{ij} \ln t_{ij} - \sum_{i=1}^{N} \theta_{j} \ln t_{i1}, j = 2, \ldots, N \]  \hspace{1cm} (10b)

where \( t_{ij} \) need not equal \( t_{ji} \). Thus, the MR approximations derived in B-B under SBTC can also be derived under ABTC (and they are the same). Thus, the MR approximations derived in B-B under SBTC also work under ABTC, as the Monte Carlo results in Table 2 of Bergstrand, Egger and Larch (2007) confirm.