Review Sheet for Final Exam

Standard disclaimer: The following represents a sincere effort to help you prepare for our exam. It is not guaranteed to be perfect. There might well be minor errors or (especially) omissions. These will not, however, absolve you of the responsibility to be fully prepared for the exam. If you suspect a problem with this review sheet, please bring it to my attention. Beware also that I will update this thing without warning if I think of something to add/change on it.

Format: The exam will be closed book and take place Thursday December 18 from 8-10 AM in Hayes-Healy 229. There will be a review session on Wednesday Dec 16 from 4-5 (or later, depending on questions) in Hayes-Healy 125. If you'd like to see an example of "thus and such" at the review session, it helps a lot to have a specific problem in hand (e.g. dig one up from a textbook) to work on.

As on the first exam, problems will fall into three categories

- Statements of definitions and important theorems.
- Binary 'true/false' type problems. You will be required to provide *specific* counterexamples to false items and perhaps answer other questions about true items.
- Computational and proof-type problems.

There's a lot of latitude in the last category, but I'd like to keep the focus more on thoughtful computation than rigorous proof. Any proofs will be straightforward and short.

The exam will be comprehensive. I'll give priority to material not covered on the midterm, but I'll be unhappy if you forget, say, how to tell whether something's a subspace or the definition of linear transformation! In addition to the material covered on the midterm (see the review sheet for the midterm for this), you should know about the following.

Things to know:

definitions and statements. Determinant function, eigenvalue, eigenvector, eigenspace, diagonalizable operator, characteristic polynomial of a linear operator, invariant subspace, Cayley-Hamilton Theorem, inner product, dual space of a vector space, orthogonal vectors and subspaces, orthogonal complement of a subspace, adjoint of a linear operator, orthogonal projection of a vector onto a subspace.

knowledge useful for answering true/false questions. Basic properties of determinants, inner products, eigenvalues and eigenvectors. Relationship between determinants and invertibility, independence, etc. Criteria for diagonalizability of a linear transformation. Examples of linear transformations with repeated or complex eigenvalues.

computational skills. Finding orthogonal bases and orthogonal projections. Computing least squares solutions of linear systems. Computing determinants–remember there are (at least) 2 ways to do this. Finding characteristic and minimal polynomials. Finding eigenvalues and eigenvectors. Finding the polynomial $p_{\mathbf{v}}$ associated to a vector $\mathbf{v}$. Diagonalizing linear transformations.

Things that won't be on the exam: I will not ask about permutations or about the formula for the determinant function in terms of permutations. Though it might be convenient for you to use cofactor expansion to evaluate some determinant, I won't demand it, and I certainly won't ask you to employ the cofactor formulas for the inverse of a matrix or the solution of a linear system.

Other resources: Please keep in mind that in addition to your lecture notes and textbook, there are solutions to each homework assignment. There are some other linear algebra textbooks on reserve in the math library. Beyond the other problems in our textbook, I recommend Bretscher's book as a source for lots of nice, mostly straightforward problems. Unfortunately, however, Bretscher has gone missing. If you've inadvertantly walked off with Bretscher, please bring it back. Regardless, I'll try and put an alternative (probably one by Anton) on reserve in its place.