Review Sheet for Final Exam

Standard disclaimer: The following represents a sincere effort to help you prepare for this exam. It is not guaranteed to be perfect. There might well be minor errors or (especially) omissions. These will not, however, absolve you of the responsibility to be fully prepared for the exam. If you suspect a problem with this review sheet, please bring it to my attention. Beware also that I will update this thing without warning if I think of something to add/change on it.

Format: The exam will be closed book and take place Tuesday December 13 from 8-10 AM in Hayes-Healy 229. There will be a review session on Sunday Dec 11 from 8-9 (or later, depending on questions) in Hayes-Healy 125. If you'd like to see an example of "thus and such" at the review session, it helps a lot to have a specific problem in hand (e.g. dig one up from a textbook) to work on.

As on the first exam, problems will fall into three categories

- Statements of definitions and important theorems.
- Binary 'true/false' type problems. You will be required to provide *specific* counterexamples to false items and perhaps answer other questions about true items.
- Computational and proof-type problems.

There's a lot of latitude in the last category, but I'd like to keep the focus more on thoughtful computation than rigorous proof. Any proofs will be straightforward and short.

The exam will be comprehensive. I'll give priority to material not covered on the midterm, but I'll be unhappy if you forget, say, how to tell whether something's a subspace or the definition of linear transformation! In addition to the material covered on the midterm (see the review sheet for the midterm for this), you should know about the following. My (very approximate) goal is to devote 2/3 of the final to material not covered in the first half of the term.

Things to know:

definitions and statements. Determinant function, eigenvalue, eigenvector, eigenspace, diagonalizable operator, characteristic polynomial of a linear operator, algebraic and geometric multiplicity of an eigenvalue, invariant subspace, Cayley-Hamilton Theorem, inner product, orthogonal vectors and subspaces, orthogonal complement of a subspace, adjoint of a linear operator, orthogonal projection of a vector onto a subspace, least squares solution of a linear system, isometry, unitary transformation.

knowledge useful for answering true/false questions. Basic properties of determinants, inner products, eigenvalues and eigenvectors. Relationship between determinants and volume, invertibility, independence, etc. Criteria for diagonalizability of a linear transformation. Remember examples of: diagonalizable and non-diagonalizable linear operators with repeated eigenvalues, self-adjoint and unitary operators.

computational skills. Computing determinants–remember there are (at least) 2 ways to do this. Finding characteristic polynomials, eigenvalues and eigenvectors. Finding the two dimensional invariant subspace associated to a complex 'eigenvalue' for a real operator. Diagonalizing linear transformations. Computing determinants–remember there are (at least) 2 ways to do this. Finding an orthogonal basis for a subspace, an orthogonal projection onto a subspace, an adjoint operator, a least squares solution to a linear system.

Things that won't be on the exam: I will not ask about permutations or about the formula for the determinant function in terms of permutations. Though it might be convenient for you to use cofactor expansion to evaluate some determinant, I won't insist on it, and I certainly won't ask you to employ the cofactor formulas for the inverse of a matrix or the solution of a linear system. I will not ask you to remember the formula (in terms of adjoint matrices) for orthogonal projection onto a subspace.

Other resources: Please keep in mind that in addition to your lecture notes and textbook, there are solutions to each homework assignment. There are some other linear algebra textbooks on reserve in the math library. Beyond the other problems in our textbook, I recommend Bretscher's book as a source for lots of nice, mostly straightforward problems.