Homework 2

(due Friday, September 13)

Warmup problem from Hubbard×2: 1.5.1 (not collected)

Problems from Hubbard×2: 1.5.3bc, 1.5.4ab (note that e.g. 1.5.4a can be rephrased more precisely as follows: *Show that the interior* $\overset{\circ}{A}$ *of* A *is open and if* $U \subset A$ *is another open set, then* $U \subset \overset{\circ}{A}$), 1.5.5

Problems from Jones: 1-31, 1-34, 1-38

Problem 1. Recall: let $\mathbf{v}, \mathbf{w} \in \mathbf{R}^n$ be vectors such that $\mathbf{w} \neq \mathbf{0}$. The *orthogonal projection of* $\mathbf{v}$ *onto* $\mathbf{w}$ is the vector $\mathbf{v}_{\|} := \frac{\mathbf{v}\cdot\mathbf{w}}{\mathbf{w}\cdot\mathbf{w}}\mathbf{w}$. We showed in class that this is the unique multiple of $\mathbf{w}$ such that $\mathbf{v}_{\perp} := \mathbf{v} - \mathbf{v}_{\|}$ is orthogonal to $\mathbf{w}$.

Now suppose that $x, y, z \in \mathbf{R}^n$ are points such that $x \neq y$. Show that the following two properties are equivalent for a fourth point $w \in \mathbf{R}^n$.

- w is the closest point to z on the line determined by x and y.
- $w = x + (z - x)_{\|}$ where $(z - x)_{\|}$ is the orthogonal projection of $z - x$ onto $y - x$.

Finally, let $L \subset \mathbf{R}^3$ be the line determined by $(1, 1, 2)$ and $(0, -1, 3)$. Compute the (minimal) distance from $(1, 0, 0)$ to L.

Problem 2. Show that distinct lines in $\mathbf{R}^n$ intersect in at most one point. Use the definition of line from Jones (the one I gave in class).

Problem 3. Four of the following statements are false. Find them and give counterexamples. Note that in all cases, there are counterexamples in $\mathbf{R}$ (i.e. you can take $n = 1$).

(a) for all $X \subset \mathbf{R}^n$, one has $\overline{\overline{X}} = \overline{X}$.
(b) if $X \subset \mathbf{R}^n$ is open, then $\overset{\circ}{\overline{X}} = X$.
(c) if $X \subset \mathbf{R}^n$ is closed then $\overline{\overset{\circ}{X}} = X$.
(d) if $X \subset \mathbf{R}^n$ has empty interior, then X is closed.
(e) the union of finitely many closed subsets of $\mathbf{R}^n$ is closed.
(f) An arbitrary union of closed sets is closed.
(g) for all $X \subset \mathbf{R}^n$ one has $\partial X = \partial X^c$.