Homework 3

(due Friday, September 20)

Warmup problem from Hubbard×2: 1.5.15, 1.5.21

Problems from Hubbard×2: 1.5.4cd, 1.5.14.

Problem 1. Use the definition of convergenge to prove the following statements about sequences $(x_j), (y_j) \subset \mathbf{R}$ of real numbers.

- $\lim x_j + y_j = \lim x_j + \lim y_j$ (provided both limits on the right exist);
- $\lim(-1)^j$ does *not* exist. I.e the sequence $((-1)^j)$ diverges.
- $\lim \frac{1}{x_j} = \frac{1}{\lim x_j}$ (provided $\lim x_j$ exists and is not zero.

For the first and third limits, the material in Apostol volume I on limits of functions might provide you with a helpful model.

Problem 2. Prove that a subset $X \subset \mathbf{R}^n$ is closed if and only if every convergent sequence $(x_j) \subset X$ satisfies $\lim x_j \in X$.

Problem 3. Prove that the function $f : [0, \infty) \to \mathbf{R}$ given by $f(x) = \sqrt{x}$ is continuous. Note that you probably have to handle continuity at 0 separately.

Problem 4. Prove that the function $\angle : (\mathbf{R}^n - \{\mathbf{0}\}) \times (\mathbf{R}^n - \{\mathbf{0}\}) \to \mathbf{R}$ that takes two non-zero vectors $\mathbf{w}, \mathbf{v} \in \mathbf{R}^n$ and returns the angle $\angle(\mathbf{v}, \mathbf{w})$ between them is continuous. You can assume that $\cos^{-1}$ is continuous here.

Problem 5. Prove that a composition of two continuous functions is continuous (wherever it is defined).