Homework 4

(due Friday, September 27)

Warmup problem from Hubbard×2: 1.2.1, 1.2.4, 1.2.7, 1.3.7, 1.7.1, 1.6.3, 1.7.8

Problems from Hubbard×2: 1.3.4, 1.3.11, 1.3.12a, 1.6.1, 1.6.2, 1.6.4, 1.6.10, 1.7.6, 1.7.11, ~~1.7.12~~ (the last problem will reappear next week)

Problem 1. Let $\mathbf{w} \in \mathbf{R}^n$ be a vector and $T : \mathbf{R}^n \to \mathbf{R}^n$ be the mapping that sends a vector $\mathbf{v} \in \mathbf{R}^n$ to its orthogonal projection $T(\mathbf{v}) = \mathbf{v}_{\|}$ onto $\mathbf{w}$.

(a) Show that T is linear.
(b) Suppose $\mathbf{w} = (1, 2, 3)$. Find the matrix for T.
(c) Use your answer to the second part to compute the orthogonal projection of $\mathbf{v} = (1, -1, 1)$ onto $(1, 2, 3)$.
(d) Let $S : \mathbf{R}^3 \to \mathbf{R}^3$ be the transformation that sends $\mathbf{v}$ to its reflection through the line joining $(1, 2, 3)$ to to the origin $\mathbf{0}$. Assuming S is linear, find the matrix for S.

Problem 2. Let $U \subset \mathbf{R}^n$ be open and $f : U \to \mathbf{R}^m$ be a continuous function. Show that for any open $W \subset \mathbf{R}^m$, the set

$$f^{-1}(W) := \{x \in U : f(x) \in W\}$$

is also open.