Homework 5
(due Friday, October 6)

Problems from Hubbard×2: 1.7.12

Problems from Jones: 2-6 (You can use Mathematica to do your 'sketching'), 2-17, 2-22, 2-23, 2-66, 2-79, 2-80, ~~2-83, 2-84, 2-87~~ (will reassign these last probs later)

Problem 1. Let $(a, b) \subset \mathbf{R}$ be an open interval and $f, g : (a, b) \to \mathbf{R}^n$ be differentiable curves. Note that $f \cdot g : (a, b) \to \mathbf{R}$ is an ordinary real-valued function of one real variable. Show that

$$(f \cdot g)'(t) = f'(t) \cdot g(t) + f(t) \cdot g'(t).$$

This fact might be useful in problem 2-22 above...

Problem 2. Do problem 2-36 in Jones using the 'directional derivative' formula

$$Df(x; \mathbf{h}) = \lim_{t \to 0} \frac{f(x + t\mathbf{h}) - f(x)}{t}.$$

Problem 3. Do 2-39 in Jones. Note that Jones uses $Df(0; h)$ to mean the directional derivative of f at $(0, 0)$ in the direction h (see the previous problem), so his notation does not imply that f is actually differentiable at $(0, 0)$. In fact, as you're asked to show, this function isn't even continuous at $(0, 0)$.

Problem 4. Consider the complex polynomial $f(z) = z^n$ as a function $f : \mathbf{R}^2 \to \mathbf{R}^2$. Show that f is differentiable at any $z = x + iy$ and that $Df(z, h) = nz^{n-1}h$, where you should interpret the product on the right side as multiplication of complex numbers. In particular, you should show that $Df(z) : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear transformation.