**Homework 6**

(due Friday, October 13)

**Warmup problems from Hubbard×2:** 1.2.2, 1.2.7, 1.3.12, 1.3.13

**Problems from Hubbard×2:** 1.3.20, 1.8.4

**Problems from Jones:** 2-7, 2-27, 2-36, 2-51, 2-55, 2-56, 2-60, 2-61, 2-83, 2-84

**Problem 1.** Recall from class the differentiable function $F(A) = A^2$, where $A$ is an $n \times n$ matrix. In class, we derived the (unintentionally punny) formula $DF(A, H) = AH + HA$. Note that since $F(A)$ is also an $n \times n$ matrix, we can compose $F$ with itself: $F(F(A)) = A^4$, $F(F(F(A))) = A^8$ as many times as we like. Let $F^n$ denote $F$ composed with itself $n$ times.

(a) Use the chain rule to write down a formula for $DF^2(A, H)$.

(b) Use induction and the chain rule to show that

$$DF^n(I, H) = 2^n H.$$

for all $n \in \mathbf{N}$.

The moral? Things are nicer near the identity matrix.