Homework 7

(due Friday, Nov 1)

Problem 1. Do the following by hand: plot the direction field for the ODE $y'y = -t$.

Problem 2. Find an autonomous 1st order ODE $y' = f(y)$ whose solution curves are all hyperbolas. It will suffice to have f defined only for e.g. positive values of y.

Problem 3. Find explicit formulas for the solutions of the following ODES and initial value problems (which might not be expressed in the most convenient form for solving...). Also use DField to draw a direction field for each ODE, and where there are given initial values, plot the corresponding solution curves by hand on top of the direction field.

(a) $y' = e^{t+y}$;
(b) $y' = -t/y$, $y(0) = -1$ and again with $y(0) = 2$;
(c) $ty' - 4y = t^3$;
(d) $y' = y + 2te^{2t}$, $y(0) = 3$.
(e) $y' + y\tan t = \sin 2t$, $y(0) = 2$.
(f) $(t-1)y' = ty$, $y(2) = e$ (note that I changed the initial value for t on 10/30).

Problem 4. One morning in the near future, you have a life-changing epiphany: math is for losers; you, on the other hand, were meant to be a millionaire. Sure in your new calling, you decide that the best way to realize your true inner self is to begin depositing money at a constant rate into an account earning 5% annual interest, compounded continuously. Your last mathematical act will be to figure the rate (in dollars per year) at which you'll need to pour money into the account in order to self-actualize in twenty years time. That is, derive the differential equation satisfied by the amount of money in your account, impose appropriate initial conditions and solve to find (what you will come to know as) your 'personal fulfillment constant.'

Problem 5. (Extra Credit–this is a tough one). A duck sits at the center of a circular pond with radius 100 meters. A fox on the edge of the pond can run at a maximum speed of 5 meters per second. The duck would like to leave the pond without being eaten. Unfortunately, this particular duck cannot take off flying from the water. So in order to escape, the duck must reach shore at some point where the fox is not and then immediately take flight. If the duck is infinitely clever about plotting its escape route and can swim at maximum speed M as long as it likes, what minimal value must M exceed in order for the duck to escape?