Review Sheet for Exam 1

Standard disclaimer: The following represents a sincere effort to help you prepare for our exam. It is not guaranteed to be perfect. There might well be minor errors or (especially) omissions. These will not, however, absolve you of the responsibility to be fully prepared for the exam. If you suspect a problem with this review sheet, please bring it to my attention.

Time and place: the exam will take place Thursday October 17 from 12:30-2:15 in DBRT 116 (one room down from our MWF classroom). I'll devote class the day before to reviewing for the exam.

Beforehand When you arrive you should hand me a careful writeup of the statement and proof of one of the following (your choice) theorems we covered in class:

- The Bolzano-Weierstrass Theorem.
- The Extreme Value Theorem.
- The Fundamental Theorem of Algebra.
- The 'C^1 implies differentiable' theorem.
- The Chain Rule.

You are welcome to consult with your humble instructor, the book, or your notes in preparing this. So of course you're not expected to memorize or invent the proofs. However, I'd like you to present things in your own words, using complete sentences, good punctuation and grammar, etc, etc. I'll count this for 20% of your exam grade.

(Other) Parts of the exam As for the rest of the exam, there are typically three sections:

(1) *A page where I ask you to correctly state several definitions and/or named theorems.*

Terms include *length* of a vector, *dot product* of two vectors, *orthogonal projection* of one vector onto another, *ball/sphere*, *line* (through two points), *open set*, closed set, *interior point*, *boundary point*, *bounded sequence*, *convergent sequence*, *limit* (of a function), *continuity* (at a point), *linear transformation*, *matrix of a linear transformation*, *directional derivative*, *differentiable/derivative* (of a mapping), *gradient* (of a scalar-valued function), *tangent vector* (to a curve).

When stating a definition or theorem, be careful to explain your notation. For instance don't say simply *U is open if....* Instead say, *A subset $U \subset \mathbf{R}^n$ is open if...* or at least *$U \subset \mathbf{R}^n$ is open if....*

Named Theorems include all the ones listed above (except 'C^1 implies differentiable') and the *Cauchy-Schwarz and triangle inequalities.*

(2) *A true/false page where I give a list of assertions, and ask you to identify the false ones and then provide specific counterexamples to each.*

Note the word 'specific'. If I include the assertion *All real numbers have real square roots*, don't say '*negative numbers don't have real square roots*'. Say simply e.g. '-7'. And you don't have to justify that your counterexamples serve their purposes. So you don't have to say *'-7 doesn't have a real square root because it's negative and the square of a real number is always non-negative.'* Just '-7' will suffice.

(3) *Some computational or short proof problems.*

For these, I will feel free (but not obliged) to recycle problems from the homework, particularly if I ask you to prove something. Concerning computations, by now you should know (among other things) how to work with dot products to do things like compute lengths, angles and distances, orthogonal projections, etc; use the Cauchy-Schwarz and triangle inequalities to estimate lengths and distances; find the matrix of a linear transformation; compute tangent vectors to curves, gradients of scalar valued functions, Jacobian matrices of mappings, and directional derivatives; use the linear approximation of a function or mapping to estimate it; find critical points of a scalar-valued function; multiply matrices (particularly in connection with the chain rule); care for small animals while effortlessly putting large matrices in Jordan form (are you still reading this?). I'm sure I'm forgetting something here.

A couple of further points.

- Since homework problems can take a long time to solve, you might wonder how on earth you'll be able to complete your exam in an hour or so. Rest assured that I'm aware of the difference in time frame and that I take that into account when choosing exam problems. On the other hand, you should be practiced-up enough that if a problem requires you to compute e.g. a gradient, then you should recognize immediately that you need to compute some partial derivatives and how to go about doing that. It's my job to choose problems where the computations don't turn into a holy mess.
- Much of the exam should be straightforward if you're prepared for it, and if you acquit yourself well on this part, then I'm satisfied. However, I reserve the right to include a challenging problem or two just to see what you can do. On these I'm at least as interested in what you try as I am in whether you completely solve the problem.