Review Sheet for Exam 1

Standard disclaimer: The following represents a sincere effort to help you prepare for our exam. It is not guaranteed to be perfect. There might well be minor errors or (especially) omissions. These will not, however, absolve you of the responsibility to be fully prepared for the exam. If you suspect a problem with this review sheet, please bring it to my attention (bounty points are possible).

Review session: Sunday Oct 5, 5-6 PM in DBRT 125 (our Thursday room).

Time and place: the exam will take place Monday, Oct 6 in class. It will cover the material from chapters 1 and 2 of Shifrin.

Ground Rules: the exam is closed book and no calculators are allowed. All you'll need are sharp pencils and a good eraser. No pens please!

Format: I typically ask three kinds of questions:

(1) A page where I ask you to correctly state several definitions and/or named theorems.

Things that would be fair game here include *length* (or norm or magnitude) of a vector, *dot product* of two vectors, *orthogonal projection* of one vector onto another, *Cauchy-Schwarz inequality*, *triangle inequality*, *subspace*, *linear combination*, *span* of a set of vectors, *linear transformation*, *standard matrix* of a linear transformation, *invertible matrix*, *open/closed ball*, *open set*, closed set, *interior point*, *exterior point boundary point*, *limit of a function*, *continuous*.

When stating a definition or theorem, be careful to introduce your notation. For instance don't start out "*U is open if...*" Instead say, "*A subset $U \subset \mathbf{R}^n$ is open if...*"

(2) 'True/false' type questions where you're asked to identify the false statements and provide specific counterexamples to each.

A partial list of possibilities: determining whether a given subset of $\mathbf{R}^n$ is a subspace; determining whether a given function $T : \mathbf{R}^n \to \mathbf{R}^m$ is a linear transformation; determining whether a given subset of $\mathbf{R}^n$ is open or closed.

Concerning the nature of your counterexamples, note the word *specific* above. If I include the assertion *All real numbers have real square roots*, don't say '*negative numbers don't have real square roots*'. Say simply e.g. '-7'. And you don't have to justify that your counterexamples serve their purposes. So you don't have to say *'-7 doesn't have a real square root because it's negative and the square of a real number is always non-negative.'* Just '-7' will suffice.

(3) Some computational or short proof problems.

For these, I will feel free (but not obliged) to recycle problems from the homework, particularly if I ask you to prove something. Concerning vectors, by now you should know (among other things) how to work with vector additon addition and scalar multiplication, dot products, lengths, angles and distances, orthogonal projections, etc; use the Cauchy-Schwarz and triangle inequalities to estimate lengths and distances. Concerning matrices, you should be able find the matrix of a linear transformation and then use it to compute images of vectors; compute sums, scalar multiples and products of matrices and understand the relationship between these things and the associated linear transformations.

You should be able to apply the definitions of subspace and linear transformation in a proof.

You should be able to relate scalar functions to plots of level sets and graphs, relate vector functions of a single variable (parametrizations) to curves. You should be able to employ the definition of limit in fairly straightforward cases (e.g. a linear function is continuous), and use various properties of limits to either compute or refute limits for more complicated functions.

Unsolicited advice. The best things you can do to prepare are

- Make sure you thoroughly understand the solutions to all the homework problems. Even if you got full credit on a given problem, it's worth comparing your solution with mine and/or trying to write out the solution from scratch.
- Make yourself a glossary of terms and theorems and memorize them.
- Get a good night's sleep.