Review Sheet for the Final Exam

Standard disclaimer: The following represents a sincere effort to help you prepare for our exam. It is not guaranteed to be perfect. There might well be minor errors or (especially) omissions. These will not, however, absolve you of the responsibility to be fully prepared for the exam. If you suspect a problem with this review sheet, please bring it to my attention (bounty points are possible).

Review session: Wednesday, December 17 from 6:30-7:30 in DBRT 317.

Time and place: the exam will take place Thursday, December 18 from 4:15-6:15 in DBRT 317. It will be comprehensive, including questions from all parts of the semester, but emphasizing newer material, particular stuff not covered on the midterms.

Ground Rules and Format: Similar to exams 1 and 2.

Specific things to know (beyond things listed on the first two review sheets).

- **Most important statements and definitions:** *Critical point*, *local maximum (or minimum)* of a scalar-valued function; *bounded subset* and *compact subset* of $\mathbf{R}^n$; *Extreme Value Theorem*; *Orthogonal/orthonormal set*; *orthogonal projection* of a vector onto a subspace; *least squares solution* of a linear system; *Inverse function theorem*; *Implicit Function Theorem* (special case only); *smooth hypersurface* in $\mathbf{R}^n$;

- **Brief glossary:** My way of stating some of the above in class differed a bit from Shifrin's. I'd prefer you go with my statements, so here they are
  - **Extreme Value Theorem.** Let $K \subset \mathbf{R}^n$ be a compact set and $f : K \to \mathbf{R}$ be a continuous function. Then there are points $a, b \in K$ such that
  $$f(a) \le f(x) \le f(b)$$
  for all $x \in K$.
  - **Inverse Function Theorem.** Let $f : \mathbf{R}^n \to \mathbf{R}^n$ be a C^1 mapping. Suppose that $a \in \mathbf{R}^n$ is a point such that the derivative map $Df(a)$ is invertible. Then there exists neighborhoods U of a and V of $f(a)$ such that $f : U \to V$ is invertible and the inverse function $f^{-1} : V \to U$ is C^1.
  - **Implicit Function Theorem.** Let $f : R^n \to \mathbf{R}$ be a C^1 function. Suppose that $a \in \mathbf{R}^n$ is a point at which $f(a) = 0$ and $\frac{\partial f}{\partial x_n}(a) \neq 0$. Then there exist neighborhoods U of a and U' of $a' := (a_1, \ldots, a_{n-1}) \in \mathbf{R}^{n-1}$, and a C^1 function $\phi : U' \to \mathbf{R}$ such that for any $x = (x', x_n) \in U$ we have $f(x) = 0$ if and only if $x' \in U'$ and $x_n = \phi(x')$.
  - **Definition.** A set $X \subset \mathbf{R}^n$ is a *smooth hypersurface* if for any $a \in X$, there exists a neighborhood U of a and U' of $a' := (a_1, \ldots, a_{n-1}) \in \mathbf{R}^{n-1}$ such that $X \cap U$ is the graph of a C^1 function $\phi : U' \to \mathbf{R}$; i.e. after reindexing coordinates if necessary, we have $x = (x', x_n) \in X \cap U$ if and only if $x' \in U'$ and $x_n = \phi(x')$.

- **Computational skills:** Finding critical points of a C^1 function; using this together with the method of Lagrange multipliers to find the maximum and minimum values of a C^1 function on a compact set. Finding least squares solutions of a linear system, orthogonal projection (in several ways) of a vector onto a subspace, orthogonal basis for a subspace. Using Newton's method to improve a guess at a solution of a non-linear system (particularly one with the same number of equations as unknowns).

Unsolicited advice. Same as last time.