Homework 1

(due Friday, September 5)

Warmup problems (Do not turn in)

- 1.1: 1 ('geometrically' in this problem means 'draw a picture that illustrates the computation'), 12
- 1.2: 1, 2

Turn in answers only:

- 1.1: # 2
- 1.2: # 3
- Recall that a *tetrahedron* is the 3-dimensional analogue of a triangle–i.e. it's a pyramid ABCD with triangular base ABC and apex D. A tetrahedron is *regular* if each of its 6 edges (e.g. AB, BC, etc) has the same length. The *centroid* of a tetrahedron is the average (A+B+C+D)/4 of its vertices. Consider here the tetrahedron with vertices $(1, 1, 1, -1), (1, 1, -1, 1), (1, -1, 1, 1), (-1, 1, 1, 1)$. You can check for yourself that this tetrahedron is regular. Find it's centroid, and compute the angle between any two line segments joining distinct vertices to the centroid. Is the angle acute or obtuse?

Turn in full solutions.

- 1.1: #4, 5, 13
- 1.2: # 5, 8, 11, 14, 17, 24

  Remark: In problem 24, you should begin by giving a good definition of 'altitude' of a triangle—i.e. *'The altitude of the triangle ABO through the point A is the set of points C such that ...'* using vector operations, dot products or some such. If it's not clear from your definition that the altitude is a line, then you should explain why it really is a line. You can take for granted that each pair of altitudes meets in a single point (in particular no two are parallel).

Extra Credit. Turn in to me separately.

- Let $C := \{(x_1, x_2, x_3) \in \mathbf{R}^3 : 0 \leq x_j \leq 1\}$ be the unit cube in $\mathbf{R}^3$ and $B(\mathbf{0}, 1) := \{\mathbf{x} \in \mathbf{R}^3 : \|x\| < 1\}$ be the unit ball. Find (with proof) the center $\mathbf{a} \in \mathbf{R}^3$ and radius $r > 0$ of the largest ball $B = B(\mathbf{a}, r)$ such that $B \subset C$ but $B \cap B(\mathbf{0}, 1) = \emptyset$.