Homework 11
(due Friday, Nov 21)

Warmup (don't turn in).

5.2:: 1agi, 3, 6

Turn in answers only.

5.1: 1a-g,k,l (It's enough to identify the sets which are not compact and state specifically whether the set fails to be closed, bounded or both.)

5.2: 1bejk

Turn in full solutions.

5.1: 8 (also, is the nearest point always unique?)

5.2: 2, 5, 12

Problem 1. The extreme value theorem holds for functions $f : S \to \mathbf{R}$ satisfying three hypotheses: the domain $S \subset \mathbf{R}^n$ of f is both closed and bounded, and the function f is continuous. Give examples illustrating the necessity of these hypotheses. I.e. for each hypothesis, give a specific example of a function $f : S \to \mathbf{R}$ satisfying the other two hyptheses, yet failing the conclusion of the extreme value theorem.

Problem 2. Show that in $\mathbf{R}^n$ the intersection of a closed set and a compact set is compact. (You may of course use what we learned earlier about intersections of closed sets.)

Problem 3. Let $V \subset \mathbf{R}^n$ be a subspace. In class I showed that if $\mathbf{v}_1, \ldots, \mathbf{v}_k \in V$ span V and $\mathbf{w}_1, \ldots, \mathbf{w}_\ell \in V$ are independent, then $\ell \leq k$. This implies in particular that if both sequences of vectors are bases for V, then $k = \ell$. Show conversely that if $k = \ell$, then both sets of vectors are bases. That is, a basis is the same thing as a maximal linearly independent set or a minimal spanning set.

Problem 4. Let $V \subset \mathbf{R}^n$ be a subspace. Recall from class that $\dim V + \dim V^\perp = n$. Suppose that $\mathbf{v}_1, \ldots, \mathbf{v}_k$ is a basis for V and $\mathbf{v}_{k+1}, \ldots, \mathbf{v}_n$ is a basis for $V^\perp$. Use the previous problem to show that $\mathbf{v}_1, \ldots, \mathbf{v}_n$ is a basis for $\mathbf{R}^n$.