Homework 2

(due Friday, September 12)

Turn in answers only:

- Find the distance between the point $(3, 1, 2)$ to the line joining $(1, 2, 1)$ to $(-1, 4, 0)$. Note that this line does *not* pass through the origin.
- 1.3: 1 (identify only the imposters, giving specific counterexamples showing that each one fails one of the three criteria in the definition of subspace).
- 1.4: 5, 15

Turn in full solutions.

- 1.2: 12
- 1.3: 4, 6, 8, 9
- 1.4: 3,
- Let $\mathbf{y} \in \mathbf{R}^n$ be a fixed, non-zero vector. Let $T : \mathbf{R}^n \to \mathbf{R}^n$ be given by $T(\mathbf{x}) = proj_{\mathbf{y}}(\mathbf{x})$.
  (a) Show that T is linear.
  (b) Find the matrix for T if $\mathbf{y} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \in \mathbf{R}^2$ and then use it to compute the orthogonal projection of $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ onto $\mathbf{y}$.
  (c) Find the matrix for the linear transformation $R : \mathbf{R}^2 \to \mathbf{R}^2$, where $R(\mathbf{x})$ is the reflection of $\mathbf{x}$ about joining $\mathbf{0}$ to $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$. You can take for granted that R is linear. This is simpler if you take advantage of the previous part of this problem. Use your matrix to compute $R(\begin{bmatrix} 3 \\ 4 \end{bmatrix})$.

Extra Credit Use the vector operations we've developed in class to prove *Heron's formula* for the area of a triangle: if $a, b, c > 0$ are the side lengths of a triangle, then it's area A is given by

$$A^2 = s(s-a)(s-b)(s-c),$$

where $s := \frac{1}{2}(a + b + c)$ is called the *semi-perimeter* of the triangle. Incidentally, 'base times height' is a fine definition of A here.