Homework 3
(due Friday, September 19)

Warmup problems (Do not turn in)

- 1.4: 1

Turn in answers only:

- 1.4: 13, 16 (in part c, assume the plane contains the *top* edge of the tetrahedron), 17, 18 (don't worry about the conjecture).
- 2.1: 1, 9, 11

Turn in full solutions.

- 1.4: 8abc, 11 (give the proofs–see me in office hours if you don't know how to do an induction proof), 20, 21
- 2.1: 2, 3 (assume the radius $a = 1$), 4 (assume $b = a = 1$), 12ab
- Given a point $\mathbf{a} \in \mathbf{R}^n$ and a real number $r > 0$, Shifrin uses $B(\mathbf{a}, r) := \{\mathbf{x} \in \mathbf{R}^n : \|\mathbf{x} - \mathbf{a}\| < r\}$ to denote the *open ball* of radius r about $\mathbf{a}$. For two such balls $B(\mathbf{a}, r)$ and $B(\mathbf{b}, s)$, one has the following tricotomy.
  (a) If $\|\mathbf{a} - \mathbf{b}\| + r \leq s$ then $B(\mathbf{a}, r) \subset B(\mathbf{b}, s)$.
  (b) If $\|\mathbf{a} - \mathbf{b}\| \geq s + r$ then $B(\mathbf{a}, r) \cap B(\mathbf{b}, s) = \emptyset$.
  (c) Otherwise $B(\mathbf{a}, r)$ intersects both $B(\mathbf{b}, s)$ and the complement of $B(\mathbf{b}, s)$.
  Prove the first two of these statements.
  Remember in both parts that your argument should begin '*Given* $\mathbf{x} \in B(\mathbf{a}, r)$ ...', and that the triangle inequality is your friend.

Remark Up to three extra points for generating mathematica plots that verify your answers to problems 2, 4 and 9 in section 2.1.