Homework 4

(due Friday, September 26)

Turn in answers only:

- 2.2: 1 (Do the following in each part. If the given set is open, just say so. If not, give a point in the set that is not an interior point. Similarly, if the set is closed, say so. If not, give a boundary point that is not contained in the set.)

Turn in full solutions.

- Let $S \subset \mathbf{R}^n$ be a set
  (a) Show that $\overset{\circ}{S}$ is open. (Perhaps the hardest thing here is to see that this could actually be an issue)
  (b) Show that $\overline{S}$ is closed.
  (c) Show that ∂S is closed.

  Note that the last two parts are easier if you show that the complementary sets are open.

- 2.2: 5, 7 (I covered the intersection of open sets in class, so no need to repeat this part), 9

- The assertions in 2.2:# 7 both hold just as well for unions and intersections of finitely many (i.e. not just two) open/closed sets. Which ones work for unions and intersections of *infinitely* many open/closed sets? No proofs necessary, but give counterexamples to false statements.

- 2.3: 2 (Use the definition of limit to do this)

- An *affine* transformation $f : \mathbf{R}^n \to \mathbf{R}^m$ is the composition of a linear transformation with a translation. Specifically, a function $f : \mathbf{R}^n \to \mathbf{R}^m$ is *affine* if it is given by $f(\mathbf{x}) := T(\mathbf{x}) + \mathbf{b}$ for some linear transformation $T : \mathbf{R}^n \to \mathbf{R}^m$ and some fixed vector $\mathbf{b} \in \mathbf{R}^m$. Use the definition of limit to show that affine transformations are continuous.
- Consider the following functions $f : (0, \infty) \to \mathbf{R}$,
  (a) $f(x) = 1/x$.
  (b) $f(x) = \sqrt{x}$.

  For each of them use the definition of limit to prove that f is continuous at each point $a \in (0, \infty)$. In each case, give the values of δ that you come up when $a = 2$, and $\epsilon = 2$ and then again when $\epsilon = .2$. Does your argument for continuity of $\sqrt{x}$ work at the point $a = 0$?

Extra Credit. Show that a subspace $V \subset \mathbf{R}^n$ is always closed but not open unless $V = \mathbf{R}^n$.